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Mathematical modelling in primary school, advantages and challenges

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Abstract

As the importance of mathematics in the global market increased, various governments have focused on training creative and competent human resources. A country that has creative and innovative workforce in the industry and technology travels the road of development and evolution faster which leads to better education and welfare, appropriate sanitation, and generally more comfortable life. From the sciences which are the bases of technology and development is mathematics, and the most appropriate way to teach mathematics is using its application and mathematical modelling. In the present article I introduce mathematical modelling and its applications, review similarities and differences with problem solving, and consider how to introduce it in elementary levels(since I believe that mathematical modelling and applications should be taught in early years of school). I also state some advantages and challenges of introducing mathematical modelling to elementary school students (and generally to school students).The main aim of the present article is to encourage developing countries and the Third World to introduce mathematical modelling into their pedagogical systems, especially elementary schools.I believe that this could lead these nations to develop faster.

Keywords: mathematical modelling, problem solving , applications

1 Introduction e Theoretical framework

The problem-solving experiences that children typically meet in schools are no longer adequate for today's world. Mathematical problem solving involves more than working out how to go from a given situation to an end situation where the "givens,"the goal, and the "legal solution steps" are specified clearly. The most challenging aspect of problems encountered in many professions today involve developing useful ways of thinking mathematically about relevant relationships, patterns, and regularities(Lesh&Zawojewski, In press). In typical elementary schools worldwide, the teaching of early arithmetic is predominantly focused on computational proficiency. Even word problems that putatively link mathematics and aspects of the real world are often no more than thinly disguised exercises in the four basic operations (Greer, Verschaffel& Mukhopadhyay,2007) and With the increased importance of mathematics in our ever –changing global market, there are greater demands for workers who possess more flexible, creative, and future-oriented mathematical and technological capabilities, Powerful mathematical processes such as constructing, describing, explaining, predicting, and representing, together with quantifying, coordinating, and organizing data, provide a foundation for the development of these capabilities, Also of increasing importance is the ability to work collaboratively on multi- dimensional projects, in which planning, monitoring, and communicating results are essential to success (Lesh&Doerr , 2003).Thereforea new perspective, or at least a change in review, is essential. in my opinion ,introducing mathematical modelling and applications into elementary schools have positive influence to remove these problems properly.

1.1 Why include modelling at the primary level?

Opportunities to explore real-life applications make mathematics moremeaningful for students and aid in the development of important otherskills. Since applications involve the use of models and some aspects ofthe modelling process, modelling cannot be ignored in the primarycurriculum. The curriculum document from the Ministry of Education in Singapore emphasises the importance of applications and modelling:Application and Modelling play a vital role in the development ofmathematical understanding and competencies. It is important that students apply mathematical problem-solving skills and reasoningskills to tackle a variety of problems, including real-world problems.(Ministry of Education, 2006, p. 8)

1.2 Problem Solving, mathematical modelling and applications

(Stanic& Kilpatrick 1989) begin their chapter on problem solving from a historical perspective with the words “problems have occupied a central place in the school mathematics curriculum since antiquity, but problem solving has not.” (p. 1),and it was first during the 19th century that problem solving started to get gradually more attention. Nevertheless, since then, (Schoenfeld, 1992) notes that ”indeed, ’problems’ and ’problem solving’ have had multiple and often contradictory meanings through the years – a fact that makes interpretation of the literature difficult.” (p. 337),. However, following (Blum &Niss 1991) a *problem* can be defined as “a situation which carries with it certain open questions that challenge somebody intellectually who is not in immediate possession of direct methods/procedures/ algorithms etc. sufficient to answer the questions.” (p. 37),. [according to (Kaur &Dindyal, 2010)].(Polya, 1971) suggested the use of stages of the problem solving process which he named as: understand the problem; devise a plan; carry out the plan; and look back and examine the solution. This is not to suggest problem solving is a linear progression from ‘givens to goals’ but rather a cyclic process with pupils often having to backtrack to earlier stages to check information or refine strategies(Verschaffel& De Corte, 1997).A mathematical problem is considered to be either a pure problem if theproblem situation in question is embedded entirely within ‘the mathematical universe’(the mathematical domain), or on the other hand, if the problem situation addresses some other disciplines or real world situations (the extra-mathematical domain) where mathematical notation and syntax are allowed to be invoked in the process of solving the problem, the problem is called an applied problem(Bergman, 2009).

Using mathematics to solve real world problems is often called applying mathematics, and a real world situation which can be tackled by means of mathematics is called an application of mathematics. Sometimes the notion of “applying” is used for any kind of linking of the real world and mathematics(ICMI Study 14, 2002)..

The application of mathematics cannot be divorced from the use of models and the modelling process. As most application problems involve the discussion of some reality or “realistic contexts”, the use of models to access that reality becomes essential.In application problems, children very often have to deal with problems in realistic contexts which can only be made accessible and manipulable if models are used. Thus, models have an important role in making mathematics become real for students(Kaur &Dindyal, 2010).

The Oxford Dictionary of English (2nd edition revisited) provides the following five interpretations when ‘model’ is entered as the headword entry: (1) a three-dimensional representation of a person or thing or of a proposed structure, typically on a smaller scale than the original… (2) a thing used as an example to follow or imitate… (3) a simplified description, especially a mathematical one, of a system or process, to assist calculations and predictions… (4) a person employed to display clothes by wearing them. (5) a particular design or version of a product… and most people can probably relate to all these meanings and understand as well as use them in everyday speech. The word *model* originates from the Italian word *modello*, which in turn originates from the Latin word *mo'dulus*, a diminutive form of *modus*, which translates to *measure* or *size*. In scientific work and debate *a model* is often equated with some sort of representation of an object, a phenomena or and idea. As is evident in the five interpretation of the word *model*. Mathematical models and modelling above, it is often the case that one either discusses models as concrete models like replicas made in different sizes or illustrations of an idea, or abstract models like mental constructions or theories (NE). A naïve, direct and intuitive meaning of the notion of a mathematical model is a model in any of the meanings described above, except in the sense of (4), that are expressed using mathematical nomenclature and syntax. However, to have a scientific discussion and debate it is important to have as clear and precise definitions of the involved concepts and notions as possible. In the case of mathematical models and mathematical modelling from a mathematics education perspective this is not a trivial matter, and this chapter aims to provide an (non-exhaustive) overview of some of the aspects of the past and present debate (Bergman , 2009)

(Ogborn, 1994), quoted in Molyneux-Hodgson, Rojano, Sutherland and (Ursini, 1999, p. 176), describes modelling in general terms as “thinking about one thing in terms of simpler artificial things”. In mathematics education research these ‘simper artificial things’ most of the time is mathematical vocabulary and syntax. (Lingefjärd, 2006), drawing on (Swetz and Hartzler, 1991) does just this: “Mathematical modelling can be defined as a mathematical process that involves observing a phenomenon, conjecturing relationships, applying mathematical analyses (equations, symbolic structures, etc.), obtaining mathematical results, and reinterpreting the model.” (p. 96)(Ang, 2009) argued that mathematical modelling can be thought of “as a process inwhich there is a sequence of tasks carried out with a view to obtain a reasonable mathematical representation of the real world”. Some mathematics educators define mathematical modelling as the process of “using the power of mathematics to solve real-world problems” (Hebborn, Parramore& Stephens, 1997, p. 42).

Despite the many differences in opinions among researchers on the term “mathematical modelling”, one common feature that occurs prominently throughout the diverse opinion of mathematical modelling can be identified: mathematical modelling involves real-life problem(Kaur &Dindyal, 2010)

Model eliciting activity is defined as a problem solving activity constructed using specific principles of instructional design in which students make sense of meaningful situations,and invent, extend, and refine their own mathematical constructs. In other words, while the traditional problem-solving goal is to process information with a given procedure, model eliciting is the process itself. The purpose of the process is for students to take their model elicited through solving the original problem and apply it to a new problem(Kaiser & Sriraman,2006).

1.3.Problem Solving and Modelling – Similarities and Differences

Stillman makes an excellent and meaningful distinction between application and modelling. She states that In mathematical applications the task setter starts with mathematics and reaches out to reality. A teacher designing such a task is effectively asking: Where can I use this particular piece of mathematical knowledge? This leads to tasks that illustrate the use of particular mathematics content. They are a useful bridge into modelling but are not modelling in themselves. (p. 305) With mathematical modelling on the other hand, the task setter starts with reality and looks to mathematics before finally returning to reality to judge the usefulness and desirability of the mathematical model for description or analysis of a real situation. (p. 306), (Ministry of Education. 2006b).problem solving is usually defined with respect to the problem solver and the process of problem solving involves a search for a means to solve the problem, usually with a focus on correct procedures and correct solutions. Whereas in modelling, the nature of the tasks posed is now the focus so that appropriate tasks require interpretation of the information and interpretation of the desired outcomes. This is best achieved in cooperative groups of pupils as they design and identify flaws in proposed models, understand limitations, as well as test and revise the model they choose for the task.Problem solving and modelling are complex processes but to support pupils’(Zawojewski, 2007) problem solving and modelling efforts, approaches have been developed to guide student thinking and promote metacognitive processes(Kaur &Dindyal, 2010).the link between problem solving and modelling as:(Zawojewski, Lesh, & English, 2003)

• teams working on problem situations;• partitioning a complex situation into parts;
• communicating information; and• planning, monitoring and assessing immediate results

1.4. a set of stages for modelling that could also be used to assist pupils

1. observing a phenomenon and delineating the problem;2. conjecturing the relationships among factors and interpretingthem mathematically(mathematizing);3. applying appropriate mathematical analysis to the model; and4. obtaining results and reinterpreting them in the context of thephenomenon (p. 3). (Swetz&Hartzler, 1991)

The modelling process described by (Verschaffel, Greer and De Corte 2000) seemsmore appropriate. These authors described the following phases in themodelling process in primary schools:

> 1. understanding the situation described;2. constructing a mathematical model that describes the essence ofthose elements and relations embedded in the situation that arerelevant;3. working through the mathematical model to identify what followsfrom it;4. interpreting the outcome of the computational work to arrive at apractical situation that gave rise to the model;5. evaluating that interpreted outcome in relation to the originalsituation; and6. communicating the interpreted results. (p. xii)

1.5.cooperative learning and mathematical modelling

Modelling activities would require classroom discourse and organisation of student groups that are orthogonal to independent work or listening to lecture-style explanationsAnderson in her chapter:Collaborative problem solving as modelling in the primary years ofschooling draws some similarities and differences between problemsolving and modelling. She draws on appropriate examples and establishes that collaborative problem solving which requires the useof processes such as questioning, analysing, reasoning and evaluatingto solve particular tasks mirrors mathematical modelling. Teachersthemselves must experience mathematical modelling so as to understandthe needs of their students engaged in mathematical modelling. Ng inher chapter: Initial experiences of primary school teachers withmathematical modelling suggests that more scaffolding was necessary toease teachers into the implementation of such tasks in the primaryclassrooms, particularly in nurturing mindset change of teachers towardsaccepting multiple representations of a problem, diverse solutions, andwhat constitutes as mathematical in the representations.

1.6 .Word problems

In common teaching practice the habit of connecting mathematics classroom activities with everyday-He experience is still substantially delegated to word problems. But besides representing the interplay between formal mathematics and reality, word problems are often the only means of providing students with a basic sense experience in mathematisation and mathematical modelling. Recent researches have documented that the practice of word problem solving in school mathematics promotes in students the exclusion of realistic considerations and a "suspension" of sense-making, and rarely reaches the idea of mathematical modelling and mathematisation (see Verschaffel et al., 2000, for a review of these studies). Several studies point to two reasons for this lack of use of everyday-life knowledge: textual factors relating to the stereotypical nature of the most frequently used textbook problems, and presentational or contextual factors associated with practicesenvironments and expectations related to the classroom culture of mathematical problem solving. Furthermore the use of stereotyped problems and the accompanying classroom climate relate to teachers' beliefs about the goals of mathematics education. This indicates a difference in views on the function of word problems inmathematics education. Researchers relate word problems to problem solving and applications, while student-teachers (and probably teachers in general) see word problems as nothing more, and nothing less, than exercises in the four basic operations which also have a justification and suitable place within the teaching of mathematics, though certainly not that of favoring mathematical modelling (Blum &Niss, 1991).

Word problems (also called story problems or verbal problems) have along history in mathematics teaching, especially at the primary level.Verschaffel, Greer and De Corte (2000) describe word problems asverbal descriptions of problem situations, typically presented in a schoolcontext, wherein one or more questions are raised, the answer to whichcan be obtained by the application of mathematical operations tonumerical data available in the problem statement. These problems aregenerally used by teachers to test for higher order thinking skills and areusually perceived as being harder by the students. Word problemsdescribe some kind of reality or a realistic context for solving problems.It is understood that the realistic aspect is from the perspective of thestudent who will be engaged in solving the problem.

Word problems or story problems involve some type of application of mathematics, Since applications involve the use of models and some aspects of the modelling process, modelling cannot be ignored in the primary curriculum(Kaur &Dindyal, 2010)Therefore; Word problems are generally used for teaching about theapplications of mathematics and they provide an excellent avenuefor primary students to engage in modelling activities.(Kaur &Dindyal, 2010)

(Reusser&Stebler, 1997), identified some contextual rules andassumptions on the part of students that seem to influence their decision,to solve mathematical problems:

- Assume that every problem presented by a teacher or in a textbook make sense.
- Do not question the correctness or completeness of problems.
- Assume that there is only one "correct" answer to every problem.
- Give an answer to every problem presented to you.
- Use all numbers that are part of the problem in order to calculate the solution.
- If a chosen mathematical operation works out without remainder (evenly), you are probably on the right track.
- If a problem is perceived to be indeterminate, equivocal, or unsolvable, go for an obvious interpretation given the information in the problem text and your knowledge of mathematical operations.
- If you do not understand a problem, look at key words, or at previously solved problems in order to determine a mathematical operation.

1.7.Some points to note by teachers about word problems used as modelling tasks include the following:(Dindyal, 2009)

1. A context that is unfamiliar can become a major hurdle for a child who wants to model the situation described in the problem. Accordingly, it is important to start with modelling situations that are not too complex for the child.2. Word problems are heavily dependent on the language used in the text to describe the reality or the realistic context. The semantics can become a major obstacle for children getting a good grasp of the reality described in the problem text. This may turn off children with poor language background or paint a different picture for some children. 3. In the solution process, it is important that the teacher does not direct the students to the type of mathematics that they will be using while modelling the problem situation. It is better to let students use mathematical procedures that they are more confident to use.4. If word problems of a similar nature are repeated for the same group of children then we run the risk of *routinising*the modelling procedure which defeats the purpose of using the word problems in the first place. These problems can become disguised practice problems.5. Too many word problems can lead to a general apprehension on the part of even very highly motivated children. Also, if the problems used in class do not match the ability level of the children, then they may get uninterested and we may run the risk of trivialising the modelling and application perspective that we wish to highlight for them.6. Since modelling activities invariably involve some kind of report, it is advisable to inform students about the potential audience of the report. Students should be advised to write a statement of some sort at the very end of the solution process.

1.7.1. a classification of word problems from a modelling perspective (Galbraith &Stillman, 2001).

Injudicious problems. These are problems which are somewhat detached from reality. For example:

One person can dig a hole in the ground in 6 hours. 10 persons working at the same rate can dig thehole in *how much time?*

Although, it is possible to find a numerical answer to this problem, it does not make much sense for 10 persons to be digging the same hole at the same rate and at the same time.

Context-separable problems. In these problems the context is quite artificial and can be stripped away to expose a purely mathematicalproblem. For example:

Kevin goes to the shop with 10 coins, which amongst others includesome 10-cent, 20-cent, and 50-cent coins. What is the largest amountof money that he can spend in the shop?

The context of going to the shop is not really important and can beignored. We can simply ask for the largest amount of money that can bespent.

Standard application problems. In these problems the mathematics iscontext-related and the situation is realistic. However, the procedure isstandard as the solver is cued to some essential information. Forexample:

One bus can sit 30 passengers. Find the number of buses required totake 250 persons to a shopping mall (no person can be left out).

There is a cue that no person can be left out. Quite often the reason whysuch problems are set is that we wish children to understand that nobodycan be left out.

Modelling problems. These are typical modelling problems in which no mathematics appears in the problem statement, where the formulation of the problem, in mathematical terms, must be supplied by the modeller. For example:

Tom wishes to find out how many apples are eaten by the students in his school in a month. Explain how he can do so

Modelling problems at the primary level can also be classified according to the degree to which some structure is provided in the problem. From this perspective, a broad classification of word problems may include three types of problems: structured problems, semi-structured problems and unstructured problems. Description and an example of each follows: (Kaur &Dindyal, 2010)

Structured problems: Structured problems are the traditional word problems in which a plausible real context is described and all necessary information is provided. The students do not have to collect any data or make any type of measurements. They already know what are the numbers involved. The problem is closed as the data provided in the problem leave little room creative responses.

Tom has 24 marbles. Jerry has 18 marbles. How many more marbles than Tom does Jerry have?

Semi-structured Problems: In this type of problems, a real-life context is described and some data are usually provided, generally in a table.

Students have to interpret the given data to complete the modelling task. The questions are open-ended and students have the opportunity to provide some creative responses .

Unstructured Problems: This type of problems is called as typical modelling problems by Galbraith and Stillman (2001). These are modelling problems in which no mathematics appears in the problem

statement, where the formulation of the problem, in mathematical terms, must be supplied by the modeller.

Find the total number of coins that students in your school carry on a particular day.

This problem is quite different from the other problems and will be more demanding on the average child as there is little support or structure provided for solving the problem. Some estimation need to be made and some data need to be collected. Students can model such problems in various ways and can provide creative responses.

Moreve rmodelling tasks in primary school can be of generalization type ,mental visualization

2. Discussion and Conclusion

In the past couple of decades, children's problem solving has engaged them in situations where the “givens,” the “goals,” and the “legal” solution steps have been specified clearly; that is, the *interpretation processes* for the child have been minimized or eliminated. The difficulty for the solver is simply working out how to get from the given state to the goal state. The solutions to these problems are usually brief answers obtained from applying a previously taught solution strategy, such as “guess and check,” or “draw a diagram.” Furthermore, although these problems may refer to real-life situations, the mathematics involved in solving them is often not real world and rarely do the problems provide explicit opportunities for learners to generalize and re-apply their learning (English &Lesh, 2003). While not denying the importance of these problem experiences, they do not address adequately the knowledge, processes, and social developments that students require in dealing with the increasingly sophisticated systems of our society. Mathematical modelling activities, in the form of meaningful case studies for children, provide one way in which we can overcome this inadequacy

We want all students involved in rich mathematical tasks to believe in themselves as mathematical thinkers, and to be able to build their knowledge, skill, and identities as successful mathematical students. To do so, requires that teachers and students develop shared expectations for participating in mathematical discussions and that teachers provide clear and explicit prompts and modelling of mathematical practices associated with argumentation.

Modelling aims, among other things, at providing students with a better apprehension of mathematical concepts, teaching them to formulate and to solve specific situation-problems, awaking their critical and creative senses, and shaping their attitude towards mathematics and their picture of it (ICMI Study 14, 2002)

Among the problems that are given to students the unstructured modelling problems are certainly the most demanding type of activity as they involve some type of data collection and analysis. Such problems may not be within the reach of all pupils, although there may be some exceptions. It is better for teachers to start with the structured modelling problems and then proceed to the semi-structured modelling problems eventually leading to the unstructured modelling problems for those students who are more able (Kaur &Dindyal, 2010)

We stress that the process of bringing *"the real world into mathematics"* by starting from a student's everyday (Bonotto, 2001)life experience, is fundamental inschool practice for the development of new mathematical knowledge. Howeverit turns out to be necessary, but not sufficient, to foster for example apositive attitude towards mathematics, intended both as an effective deviceto know and critically interpret reality, and as a fascinating thinking activity.We contend that these educational objectives can only be completely fulfilledif students and teachers can bring mathematics into reality. In other words, besides *"mathematising everyday experience"* it is necessary to be *"everydaying mathematics"* (Bonotto, 2001). This can be implemented in a classroom by encouraging students to analyse*' mathematical facts'* embedded in appropriate *'cultural artefacts*'; there is indeed a great deal of mathematics embedded in everyday life (Bonotto, 2007).

According to(Blum &Niss, 1991) the following five principle arguments are invoked in the literature for the inclusion of mathematical modelling in mathematics education: 1. *The formative argument* focuses on the students' development of general capabilities and attitudes like fostering explorative and creative problem solving competencies as well as open-mindedness and self-confidence; 2. *The 'critical competence' argument* emphasises the importance to make students aware of the use and possible misuse of mathematics in society; 3. *The utility argument stresses* the use of mathematics in extra-mathematical professional and private domains; 4. *The 'picture of mathematics' argument* aims to provide the students with a rich facetted picture of mathematics as a science and an integral part of society and culture;5. *The 'promoting mathematics learning' argument* emphasising instrumental aspects of modelling in the students learning of mathematical knowledge.

Some other points to note by teachers about word problems used as modelling tasks include the following : (Dindyal, 2009)

1. A context that is unfamiliar can become a major hurdle for a child who wants to model the situation described in the problem. Accordingly, it is important to start with modelling situations that are not too complex for the child.2. Word problems are heavily dependent on the language used in the text to describe the reality or the realistic context. The semantics can become a major obstacle for children getting a good grasp of thereality described in the problem text. This may turn off children withpoor language background or paint a different picture for somechildren.3. In the solution process, it is important that the teacher does not directthe students to the type of mathematics that they will be using whilemodelling the problem situation. It is better to let students usemathematical procedures that they are more confident to use.4. If word problems of a similar nature are repeated for the same groupof children then we run the risk of *routinising*the modellingprocedure which defeats the purpose of using the word problems inthe first place. These problems can become disguised practiceproblems.5. Too many word problems can lead to a general apprehension on thepart of even very highly motivated children. Also, if the problemsused in class do not match the ability level of the children, then theymay get uninterested and we may run the risk of trivialising themodelling and application perspective that we wish to highlight forthem.6. Since modelling activities invariably involve some kind of report, itis advisable to inform students about the potential audience of thereport. Students should be advised to write a statement of some sort at the very end of the solution process.

For a real possibility to implement this kind of classroom activities, there also needs to be a radical change on the part of teachers. They have to try: i) to modify their attitude to mathematics that is influenced by the way it was learned; ii) to revise their beliefs about the role of everyday knowledge in mathematical problem solving; iii) to see mathematics incorporated into the real world as a starting point for mathematical activities in the classroom, thus revising their current classroom practice, and iv) to investigate the mathematical ideas and practices of the cultural, ethnic, linguistic communities of their pupils. Only in this way can a different classroom culture be attained. Finally a teacher has to be ready to create and manage open situations, that are continuously transforming and of which he/she cannot foresee the formal evolution or result (Bonotto, 2007) a) The systemic inertia barriers; barriers related to teachers' habits and beliefs, as well as teaching skills of teachers and teacher educators, but also the power balance within the subject regarding for example basic skills v. problem solving, or pure v. applied mathematics. b)The real world barrier; introducing the real world in the mathematics classroom makes the already demanding task of teaching mathematics (mathematics in the sense as a pure abstraction) even more demanding and complicated. In addition, is modelling 'proper mathematics'?c) The limited professional development barrier; a changing curriculum calls for professional development of practicing teachers through for instance adequate in-service courses. In addition teacher education programmes must be up-to-date and include aspects of the teaching and learning of mathematical modelling. Generally such courses and programmes are rare. d) The role and nature of research and development in education; the argument is that educational research "is not well organised for turning research insights into improved practice" (p. 192). (Burkhardt, 2006).

We must know that the less frequently students work together in small groups, the higher the mathematics achievement. It seems that the excessive use of this practice does not necessarily promote mathematics achievement (Jurdak, 2009).and It would not be possible to write down a list of all the skills which may be needed in developing mathematical models. They are many and varied: some of them may be described as intuitive, others come from long experience and practice, and some could be described as just plain common sense

The benefits of modelling introduction at schools, are fallows:

Mathematical modelling allows students to connect classroom mathematics to the real world, showing the applicability of mathematical ideas (Zbiek& Conner, 2006; Stillman, 2009). Given a real-world problem, students need to understand the real-world situation and make assumptions in order to devise a mathematical method to tackle the problem. Thus, mathematical modelling deepens students' understanding and enriches students' learning of mathematics. When students work in groups to tackle the problem, they also develop important 21st century skills such as collaborative learning skills and metacognitive skills (Tanner & Jones, 2002; McClure &Sircar, 2008).

I believe that immersing students in situations which can be related to their own direct experience and are more consistent with a sense-making disposition, allows them to deepen and broaden their understanding of the scope and usefulness of mathematics as well as learning ways of thinking mathematically that are supported by mathematising situations. Furthermore in this way we can design better opportunities for children to develop mathematical knowledge that is wider than they would develop outside of school, but that also preserves the focus on meaning found in everyday situations (Bonotto, 2007)

students learn to ask certain types of questions that can only be answered by means of mathematics (Swan, Turner, Yoon&Muller , 2007)

Modelling facilitates the development of competencies in the use of symbolic and formal mathematical systems. Powerful opportunities arise for students to strengthen their understanding of such systems by:

• forging connections between contexts and the formal mathematical expressions related to those contexts;• motivating the study of applications of abstract mathematical formulations.(p280) (Swan, Turner, Yoon&Muller ,2007)

Modelling is a powerful promoter of meaning and understanding in mathematics. When presented with problems set in some real world context, students formulate questions about the context and think about the usefulness of their mathematical knowledge to investigate the questions. They are inmiediately encouraged to connect their mathematical knowledge with the external context. Mathematical thinking is promoted, and reasoning skills are exercised, as students seek to make those connections.(p282) (Swan, Turner, Yoon&Muller , 2007)

Modelling encourages reasoning through the complementary processes of simplification and elaboration. *Simplification* involves: analysing the elements of a problem situation; identifying features that are more or less important; making assumptions that might assist in making the problem more amenable to analysis; identifying sub-problems; breaking the problem down to its essential components; expanding components and looking for suitable representations to help clarify and explore the selected components and to work towards a useful mathematisation of the problem; and defining a clear way of approaching the problems to be solved. *Elaboration* works through reviewing and refining the initial outcomes of modelling, to enable progress towards further development of a more complete model and more generally applicable solution to the original problem. These processes engage the student in long chains of reasoning. (p281) (Swan, Turner, Yoon&Muller , 2007)

In contrast to typical school problems, modelling tasks do not present the key mathematical ideas “up front.” Rather, the important mathematical constructs are embedded within the problem context and are elicited by the children as they work the modelling problem. (English & waters, 2005)

A modelling problem is a realistically complex situation where students engage in mathematical thinking (beyond that of the traditional school problem) and generate conceptual tools needed for some purpose (Lesh&Zawojewski, In press). Modelling problems foster and reveal children’s mathematical thinking thus enabling teachers to capitalise on the insights gained into their children's mathematical developments(English & waters, 2005)If we wish to establish situations of realistic mathematical modelling, in the sense of "both real-world based and quantitatively constrained sensemaking" in problem-solving activities, we haveto: i) change the type of activity aimed at creating interplay between the real world and mathematics towards more realistic and less stereotyped problem situations; ii) change students' conceptions of beliefs about and attitudes towards mathematics (this means changing teachers' conceptions, beliefs and attitudes as well); and iii) change classroom culture by establishing new classroom socio-mathematical norms (Reusser&Stebler, 1997).

In modelling, students experience concrete embodiments of new mathematical concepts. . (p284) (Swan, Turner, Yoon&Muller , 2007)From a theoretical perspective, both the help-giver and the help-receiver may benefit from sharing information, especially explanations or detailed descriptions of how to solve problems or carry out tasks. Gibing explanations may help the explainer to recognize and clarity material, recognize misconceptions, fill in gaps in his or her own understanding, internalize and acquire new strategies and knowledge, and develop new perspectives and understanding (Bargh&Schul 190; King 1992; Peterson et al.1981; Rogoff 1991; Saxe et al.1993; Valsiner 1987; Webb 1991;acoording to(Noreen &Webb,2008)..The problems allow for multiple approaches to solution and can be solved at different levels of sophistication, thus enabling all children to have access to the important mathematical content(English & waters, 2005).

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