Journal of Mathematical Modelling and Application
2013, Vol. 1, No.8, 63-67
ISSN: 2178-2423

Modeling, Mathematical Modeling and Transdisciplinarity

Lênio Fernandes Levy, PhD.
Federal University of Pará, Brazil.
leniolevy@ig.com.br

Adílson Oliveira do Espírito Santo, PhD.
Federal University of Pará, Brazil.
adilson@ufpa.br

Abstract
This article deals with the mathematical modeling and his potential interdisciplinary character. From the mathematical representation of certain objects of study related to people's daily lives, it becomes possible to think a dialogue between the mathematician and other thoughts, i.e., between mathematics and other fields of knowledge, culminating themselves often with transcendence of one's own disciplinary boundaries. In this article, we report a practice of modeling carried out during the course "Mathematical Modeling in Teaching" (class of 2002), of the Program for Postgraduate in Education in Science and Mathematics (PPGECM) of the Institute of Mathematical and Scientific Education (IEMCI) of the Federal University of Pará (UFPA). The authors of this research, whose theme is "public transportation in the city of Belém: a critical look at the quality of services provided", suggests that the practice described in the article is taken as reference in order to further exemplification and / or application Mathematical Modeling in Teaching. Based on the following question: "there has been progress in recent years in quantitative terms, the fleet of buses in the city of Belém, which is consistent with (or so higher than) the increase in the average number per month of users?", used them to modeling techniques in order to announce the trends regarding the quality of service in focus (public transportation), including the possibility of him not being available to the population so consistent. As the models show a decrease in the number of users alongside a little variation of quantitative of collective vehicles, we are led to conclude that the quality of the provision of that service (in relation to the magnitude of the space available to users inside the bus) has not been impaired.
Keywords: Modeling, mathematical modeling, transdisciplinarity.

1. Introduction

The action of modeling, as we ponder, is that whereby it intends to represent, explicitly, certain object, phenomenon or event. Certainly, it's present since the dawn of man: the cave paintings, which are among the most remote artistic works that we know, are indicative of scenes of everyday life of our ancestors, constituting, therefore, models of what they considered to be their world. Biembengut & Hein (2000) emphasize that the creation of models with a view to the representation of phenomena is inherent in man, who has always appealed to them to communicate with their peers or to prepare an action.

The model can take essential function regarding the maintenance and the comfort of its users, such the protection of the clothing, which sometimes is like a thicker skin against the elements; or the aid of glasses, that are "real" eyes for those whose eyes contain imperfections; or the defense of the weapons, that are substitutes from the jaws / fangs that we have no more – because of our physical evolution – against the danger; among other examples simulators of (and / or "inspired by") natural and social elements.

Among the possibilities for modeling, there is one that stands out for action of mathematizing aspects of an object, an event or a phenomenon. The name "mathematical modeling" is attributed to this.

From the Modern Age, despite the paradigm of fragmentation there was a strengthening of ties between mathematics and the other sciences. Bassanezi (2002) ensures that physics, astrophysics and chemistry are today quite mathematized, and that biology is following the same trajectory. This effect is less noticeable in the social sciences, where, nevertheless, strategies have been directed by the statistics. The economy is also using a robust mathematical apparatus to establish theories and market equilibriums.

> Has for some time, among various professionals, the consensus is that the expertise of specialists such as physical or engineer would be combined with competence in mathematics. Currently this pattern of thinking is being applied to different fields of knowledge - that is, the awareness about a theory or your own validation depends largely on the ability of interpretation / explanation in mathematical language.
> We cannot deny that mathematics has strongly penetrated in Economics, Chemistry, Biology, among others, in connection with the use of mathematical models, often supported, at the beginning, in the paradigms that guided the Physics - as the conservation laws and analogies consequential. Other areas such as Sociology, Psychology, Medicine, Linguistics, Music and even History begin to believe in the possibility of having their theories modeled by mathematical language (Bassanezi, 2002, p.173).

In the sphere of education – where, unfortunately, there is still largely fragmented teaching work –, the student studies math without imagining the contexts and / or the knowledge that this discipline may be related. Such ignorance will cause damage, quote, for us dwelling on in just one example, the possible difficulty as to build links between aspects of life outside of class that student and the mathematical topic “polynomials”. We emphasize that some aspects of your daily life can be represented by polynomial expressions without him imagine that possibility. He runs the risk, in addition, of not imagine such a possibility, if the education compartmentalized – which today is offered to him – continues. We conclude that the fragmented conception of contexts and / or knowledge, denoting difficulty of marching towards a complex consciousness, helps maximize the distance between school and life.

If the intent is to glimpse the complexity of the object of study, the use of modeling in school requires the establishment of links between knowledge and / or contexts sometimes isolated in its compartments. It feeds, in theory, the intellectual pleasure because the creating such links and / or because the perceived reduction of distance between subject and object.

Biembengut & Hein (2000) declare that the mathematical modeling is currently used in all sciences, contributing to the evolution of human knowledge, being, moreover, present in many everyday activities, requiring, therefore, only the existence of a problem that requires critical sense, intuition and mathematical tools. We should not, in this sense, dismissing it (the mathematical modeling) in the school context. Data related to daily urban, for example, the number of vehicles that travel in a large city and the number of its inhabitants, if we compare compare them with each other in the past decade, (they) allow, by mathematical construction and by cognitive elaboration related to some other fields of knowledge, some preview of events, making it possible measures aiming at maintaining and / or at improving the quality of life. We have here a modeling problem involving geography, health, environment, statistics and mathematics, among other disciplines.

In the following pages, we report an experience guided by mathematical modeling, which we suggest as additional reference for exemplification and / or application of this “trend in mathematics education” in college-level classes. We think the experiment gathered (and even transcended, due to reflections / critiques that it raised) mathematical, geographical, environmental, social, economic and political aspects.

The procedures were performed in March 2003, due to demand in order to obtain credit approval for the course “Mathematical Modeling in Education”, of the Program for Postgraduate (at Master level) in Education in Science and Mathematics of the IEMCI / UFPA.

The issue in focus (in other words: “the public transportation in Belém city: a critical look at the quality of services provided”) is directly targeted to the interests and needs of the community, because the largest portion of the population of Belém city uses the bus for transportation. In that case, they are predominantly persons belonging to classes less favored, regarding the socio-economic aspect. It requires, in our view, a duty to government authorities and also to entrepreneurs who have been granted the right to operate public transportation lines, in the sense that such service is available with a reasonable level of decency.

The question that guided this research was: “there has been progress, in recent years, in quantitative terms, of the bus fleet of Belém city, compatible with – or higher than – the growth of the average number of users / month?”

There were two possibilities (hypotheses) imagined before the modeling process: yes and no. If “yes”, the quality of service would be maintained (or was improving) over time, because an increasing number of public transportation vehicles equally (or more) to increase the quantity of its users correspond to conservation (or potentiation) the comfort of the clientele about the internal spatial aspect of the bus. If “not”, would have a considerable problem of social scope. Initially, we believed that the hypothesis “more realistic” was the second. The quantitative increase of users is not being counterbalanced (that was our suspicion!) for a necessary increase of the existing fleet, but by the adoption of other alternatives, like: (i) illegal transport; (ii) movement, with greater speed, the same number of authorized buses, which would offset the growing passenger demand, attitude certainly dangerous; (iii) elevation fleet of taxis and minibuses, which price of service is more costly to the citizen; (iv) Increased number of cyclists etc.

The major aim (the objective) of this research is / was creating indicative of mathematical order that would allow us to advertise trends in service (mass transit) quality in focus, which included the possibility of it not being available to the public so suited.

2. Methodology

For the treatment of data obtained in Transportation Company of Belém (CTBEL), we used the feature called “mathematical modeling”, which allows us to produce models representative of situations encountered in real life. Thenceforth, some prediction of phenomena in focus becomes possible, constituting a “north” for those wishing to concentrate on solving of future problems related to the studied events. According Biembengut & Hein:

> The creation of models to interpret the natural and social phenomena is inherent to the human being. According to Granger (1969), the model is an image that comes to mind when the rational spirit tries to understand and express intuitively a sense, trying to relate it to something already known, effecting deductions. So much so that the notion of model is present in almost all areas: Art, Fashion, Architecture, History, Economics, Literature, Mathematics etc. Indeed, the history of science is a witness to it all! The goal of a model can be explanatory, educational, heuristic, director, of forecasting, among others (Biembengut & Hein, 2000, p. 11).

Perhaps the information that have been provided us were not as reliable as we wanted, because there is not[1], in our society (the Brazilian society as a whole), a culture focused on the collection and the rigorous treatment of data. Nevertheless, we were hoping that this impact was minimal, for the purpose of not endangering significantly the outcome of our investigation.

2.1 Data tables

2.1.1 Demonstrative relating to changes in fleet of the effective transport system of the RMB (Metropolitan Region of Belém city), from December 1990 until December 2002, based on procedures performed by management of cadastral of the CTBEL:

Year												
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
1990												1,175
1991	1,177	1,180	1,184	1,209	1,207	1,218	1,222	1,224	1,220	1,222	1,216	1,222
1992	1,246	1,244	1,244	1,262	1,273	1,272	1,281	1,278	1,289	1,292	1,312	1,316
1993	1,304	1,308	1,317	1,312	1,310	1,306	1,312	1,313	1,311	1,314	1,308	1,313
1994	1,309	1,311	1,314	1,310	1,317	1,362	1,350	1,401	1,412	1,415	1,416	1,416
1995	1,426	1,436	1,439	1,442	1,461	1,451	1,446	1,465	1,462	1,474	1,500	1,504
1996	1,504	1,505	1,507	1,496	1,520	1,552	1,560	1,573	1,593	1,596	1,623	1,637
1997	1,638	1,644	1,659	1,652	1,676	1,683	1,688	1,667	1,676	1,681	1,675	1,684
1998	1,684	1,679	1,689	1,696	1,697	1,708	1,731	1,744	1,740	1,741	1,741	1,740
1999	1,740	1,744	1,744	1,738	1,741	1,728	1,719	1,735	1,735	1,729	1,734	1,718

[3] The onus of this statement is entirely ours!

2000	1,719	1,726	1,724	1,715	1,725	1,735	1,733	1,727	1,727	1,727	1,730	1,729
2001	1,728	1,726	1,719	1,721	1,704	1,708	1,721	1,720	1,721	1,730	1,712	1,720
2002	1,732	1,692	1,723	1,711	1,700	1,696	1,695	1,685	1,686	1,688	1,682	1,692

Source: CTBEL

2.1.2 Annual average of the fleet:

Year	Average of the fleet
1990	1, 175. 00
1991	1, 208. 42
1992	1, 275. 75
1993	1, 310. 67
1994	1, 361. 08
1995	1, 458. 83
1996	1, 555. 50
1997	1, 668. 58
1998	1, 715. 83
1999	1, 733. 75
2000	1, 726. 42
2001	1, 719. 17
2002	1, 698. 50

2.1.3 Equivalent passengers transported by public transportation system of the RMB (Metropolitan Region of Belém):

Year	Average of passengers Equivalent / month
1995	28, 455, 148
1996	27, 675, 841
1997	28, 126, 493
1998	28, 126, 493
1999	27, 154, 148
2001	25, 606, 294
2002	25, 305, 194

Source: CTBEL

3. Results

The data presented in Table 8.2, which expresses the average number of collective fleet of vehicles in the metropolitan region of Belém between the years 1990 and 2002, were "(re) processed" in order to feed the formula that leads to the Pearson Correlation Coefficient (r), denotative, as asserted Bassanezi (2002, p. 58), "an instrument for measuring linear correlation".

It is important to note that the functional relationships are possible to become linear, in other words, proceeding to adjustments in its structure, it becomes possible to obtain other applications (associated with the original), this time 1st degree applications. The resulting ordered pairs of actions (also inherent in the process of linearization) over initial data or initial pairs of the problem, to swell the formula of Pearson, allow us to conclude whether the original functions (nonlinear) are reliable approximations of the variables (x , y) that was available *a priori*. The more the values of "r" tend to numbers 1 or -1, the more "consistent with reality" are the (original) functions in focus. The opposite happens when the values of "r" tend to zero (0).

The results of "r" (Pearson Correlation Coefficient) we obtained, based on Bassanezi (2002, p. 58-79), were:

a) Hyperbolic Model (r = –0. 779619797);
b) Michaelis-Menten Model (r = 0. 844512293);
c) Exponential Model (r = 0. 9457295);
d) Asymptotic Exponential Model (r = –0. 947116);
e) Geometric Model (r = 0. 947881245).

We conclude that the Geometric Model, among that we had, was one whose Coefficient of Linear Correlation closest to 1 or -1. Thus, Bassanezi (2002, pp. 65-66) allowed us to construct the functional relationship indicated below:

$$Y = (1, 084. 306937)\ X^{0. 18445793}$$

Similarly, the elements of table 8.3, indicative of the monthly average of passenger versus the time (year) between 1995 and 2002 (except for the corresponding value for 2000, which had not been informed by CTBEL), led us the following models and their respective Coefficients Pearson:

a) Michaelis-Menten Model (r = –0. 647415501);
b) Hyperbolic Model (r = 0. 865949511);
c) Geometric Model (r = –0. 891063091);
d) Exponential Model (r = –0. 925626451).

With the aid of Bassanezi (2002, pp. 61-63), we find the following formula (exponential model whose linear correlation was more acceptable):

$$Y = (29, 246, 658. 04)\ e^{-0. 017088884\ X}$$

4. Discussions

The amount of data (to which we had access) on the variation “fleet x time”, whose period covered is from 1990 to 2002, is greater than the corresponding amount to changes “N. º users x time” (these latest changes concern only years 1995 to 2002, without the inherent values to 2000), which implies significant limitation to our work. Moreover, it is necessary to emphasize that there are discrepancies between the information issued by CTBEL (when our visits to that organ), with respect to Table 8.3, and those contained in the Statistical Yearbook of the Municipal Bureau of Management and Planning (SEGEP – Belém city). We decided, nevertheless, to adopt the reports that have been passed by the representative of CTBEL, and, assuming its acceptability, we conclude that:

a) The amount of public transportation vehicles, which had previously increased most noticeably, currently ranges[2] from less sensitive way (“power function” with the concavity facing downwards);
b) The number of users, who ranged in other periods with highest values, is undergoing a process of decline since 1997/1998 (decreasing exponential function).

As the models show a decrease in the number of users alongside a little variation of quantitative of collective vehicles, we are led to conclude that the quality of the provision of that service (in relation to the magnitude of the space available to users inside the bus) has not been impaired.

References

Bassanezi, R. C. (2002). *Ensino-aprendizagem com modelagem matemática*. São Paulo, Brasil: Contexto.

Biembengut, M. S.; Hein, N. (2000). *A modelagem matemática no ensino*. São Paulo, Brasil: Contexto.

[2] Take into account the year of realization of the research: 2003.