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Experiencing Mathematical Modelling in an amusement park

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Abstract
This paper is meant to be a short report of a teaching experience of modelling carried out in Italy with high school students[1] in an amusement park. Such an environment provides in fact numberless samples of curves such as parabolas, clothoids, straight lines. An amusement park is also a wonderful place where conics become visible and closer to the students' previous experience, so that learning Maths involves experimenting models on the field, and where amusement and learning do successfully join together. On such a setting students may follow all the steps of the M&A cycle, also thanks to some particular mathematical instruments and "mathematical machines", which lay a "bridge" between the domain of mathematics and the domain of reality.
This paper tries to describe the birth and the development of an original educational project, the way it was realized, and how it displays the typical features of a mathematical modelling process. This project has now become part of a set of educational workshops that the amusement park Mirabilandia offers to all schools.
At the end of the paper the authors provide their own reflections (and try to arouse the readers') on the peculiar features that modelling in such a unique environment as an amusement park can present.

Keywords: modelling; laboratory/workshop; mathematical instruments

Introduction

"What is the shape of a "loop the loop" on a roller coaster? What trajectory does the passing of a whirling carousel describe? When you look at a Ferris wheel do you really see a circle?" These are some of the questions that we tried to answer with the project Matebilandia, whose name combines the Italian words Mate-matica and Mira-bilandia. It consists of mathematical workshops[2] aimed at teaching mathematical content that are undertaken in Mirabilandia[3], an amusement park located near Ravenna (Italy).

These workshops were first tested in the spring of 2008, and only involved a hundred high school students. A further test was held the following spring, and about two hundred students from eleven different classes of high school were offered the workshops. After this second experience, the project was made available for all schools interested in Italy. The workshops have been held by the staff trained by the developers of Matebilandia, allowing the participation of thousands of students from different parts of Italy.

The idea of carrying out educational activities in an amusement park is not an absolute novelty in the international Physics-related literature (see, for example, (Bakken, 2011), (Pendrill, Lindberg, Weibull, 2005), (Pezzi, 2011)): the park can be assumed as a non ordinary classroom, fit to conduct experiments otherwise impossible in a standard school laboratory.

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[1]Students aged 14 through 19.
[2]Such workshops were conceived, designed, and actually realized by a group of teachers of Mathematics and Physics at High School, namely Lorenza Resta, Sandra Gaudenzi, Stefano Alberghi, Giovanni Pezzi, Alessandro Foschi, Lucia Paglialonga. Most of these teachers are currently teaching at the "Liceo Scientifico Torricelli" in Faenza (RA).
[3]http://www.mirabilandia.it/

Even in Italy, in 2001, some educational projects focused on Physics were held at Mirabilandia: they basically consisted in workshops aimed at studying some special features and fundamental Physics concepts evident in various park rides such as roller coasters, tower falls, Ferris wheels, carousels[4]. After some years, such an experience as this "open air classroom" seemed fit to be applied to Mathematics as well.

The project and its aims

Starting from real problems such as those you may encounter in an entertaining environment, the project takes and emotionally involves the students in mathematical activities, aimed also at providing them with a broader view of the discipline, then experienced as an efficient tool for exploring reality and figuring out mathematical models, besides remaining the remarkable legacy of a great history and culture. The educational proposals are thus designed to enable students to follow step-by-step processes of Maths modelling[5].

The Maths theme chosen was "geometrical curves", which the park is full of. The Katun, the Eurowheel (the Ferris wheel), and the carousel named "Ducks Breakfast" were the park rides considered by the project developers, then Maths modelled and lastly taken as the subject of educational activities, which will be described later on in the paper.

In details, the didactic objectives of the project can be summarized as follows:

– to show the students some samples of mathematical curves different from each other, explore their properties and characteristics and take them into consideration as mathematical models of real situations;

– to connect the curves studied to other contexts and disciplines, such as Physics, Astronomy, thus giving an interdisciplinary value to the project;

– to use instruments (Excel, TInspire, Derive, Cabri, Java or similar ones) to investigate further aspects;

– to lead the students in the analysis of a given problem, by gradually inserting further difficulties in order to have them ponder on the consequences of each step;

– to encourage the students to seek personal solutions to real problems, by promoting the " problem solving" attitude and thus fostering their critical thinking.

To sum up, the activities below proposed radically change the "when", the "how" and the "where" to do mathematics, moving from classroom activities to modelling workshops to be carried out within an amusement park, during a school excursion and following a specific working methodology.

The didactic activities

Here below there's a short description of the activities regarding the Ferris wheel "Eurowheel", the roller coaster "Katun" and the carousel "Ducks Breakfast".

Fig 1: the Perspectograph

[4]Dating from 2001 the users of such rides have been experiencing on their body the consequences of speed, acceleration, centrifugal force, weightlessness, by using traditional measuring instruments (metres and chronometers), pocket-size instruments fit for instant data recording, ad hoc built instruments and tools, such as needle accelerometers, or spring accelerometers, glasses of water, little balls, etc.

[5]As to the opportunity to teach modelling see, for instance, (Blum, Borromeo Ferri, 2009)

Fig 2 The Ferris Wheel

The Ferris Wheel project (Fig 2) [6] has the students analyze the shape of the profile, looking at it from central and lateral sides. After trying to guess the mathematical profile of the wheel, an instrument, "the Perspectograph" (Fig 1) (similar to those used by the painters of the Renaissance to convert into a two-dimension framework the view of three-dimensional objects), will be introduced to design the shape of the profile of the wheel. Once figured-out the shape of the wheel, students will have to analyze its characteristics and detect the data essential to defining the mathematical curve adopted as a descriptive mathematical model, the ellipse, thus determined by the lateral position of the observer. To validate the model, students will have to superimpose (Fig 5) the shape of the wheel with the graph of the mathematical curve drawn out with traditional instruments, such as those used by the gardeners (Fig 3) when figuring out the ellipse, and with "mathematical machines", such as the "Cross Ellipsograph" (Fig 4).

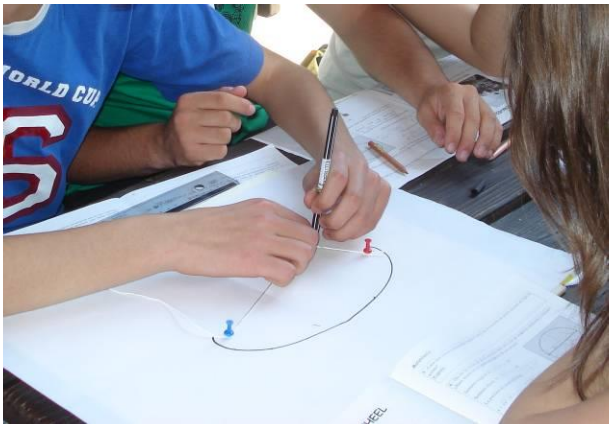

Fig 3 The "Gardener's Ellipse"

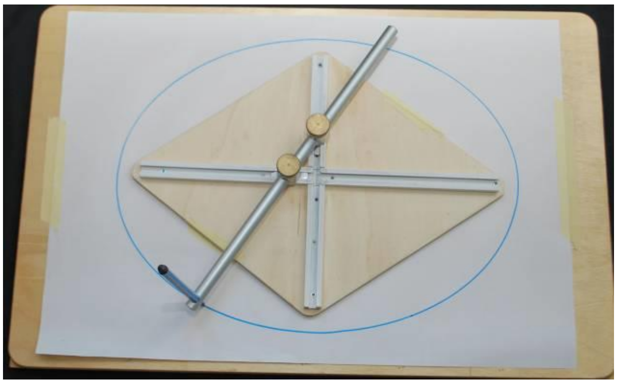

Fig 4 Cross Ellipsograph

[6]Many of the pictures here shown are taken from the book "Matebilandia: Laboratorio di matematica e modellizzazione in un parco di divertimenti" (Resta, Gaudenzi, Alberghi, 2011)

Fig 5 Superimposition of the elliptic shapes

Questions may be elicited on the perspective of the objects, simply by varying the point of view or the inclination of the observation plane being observed. By using the "Wire Perspective-Machine"[7] (Fig 6), students may analyze this problem by varying the reciprocal position of the elements (wheel, observer, observation plane) and be shown how one can generate various conic sections.

Historically, the study of perspective is linked to that of shadows cast by objects. In this project we consider the shape of the shadows produced by the wheel illuminated by a light placed at different heights. With a model consisting of a wooden plane and a metal disc illuminated by a flashlight with adjustable height (Fig 7) the various types of conic, such as ellipse, circle, parabola, hyperbole may be reproduced.

Fig 6 Wire Perspective-Machine

[7]Many of the pictures here shown are taken from the book “Matebilandia: Laboratorio di matematica e modellizzazione in un parco di divertimenti” (Resta, Gaudenzi, Alberghi, 2011)

Fig 7 Conics and shadows

The wheel is also suitable for problem solving activities, such as, for example, finding out how much colored ribbon it takes to cover the edge of the wheel using some objects chosen from among some available. The imagination of the students has provided several possible solutions, some of which have used the polygonal structure of the wheel, while by others the wheel shape has been figured out approximately with simple geometric figures, like the circumference. In the latter case the diameter of the wheel has been estimated both from the ground level (by means of trigonometry, or similarity), and by boarding the Ferris wheel with a barometric sensor.

Fig 8 Roller coaster "Katun"

Fig 9 Students riding Katun

The roller coaster Katun (Fig 8) is extremely rich in terms of features and characteristics related to Physics, engineering, technology and even Mathematics, its shape including curves, parabolas, straight lines, circumferences, clothoids, helices, etc.

Looking at the profile of the first descent of the coaster, the parabola is immediately assumed as a mathematical model for it. A photograph of the section of the rail considered provides us with the data necessary to draw out the exact curve, by using a "Parabolograph" (a parabolas drawing device) [8]. (Fig 10) The graph thus obtained is then compared to the initial picture to check the efficiency of the mathematical model adopted.

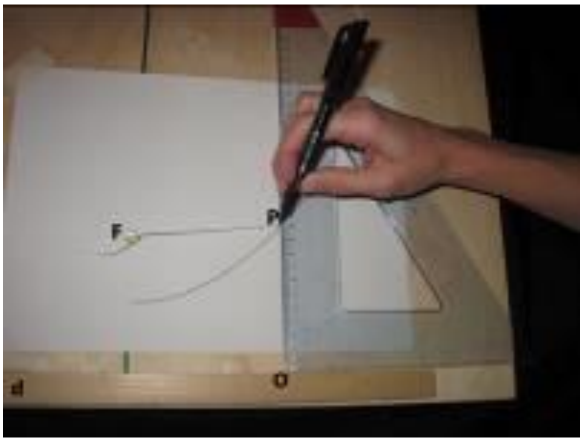

Fig 10 Parabolograph

Fig 11 The profile of the first descent of the coaster

The initial ascent of the coaster is often immediately compared to a straight line segment, with the slope being calculated in various methods and media: photographs, map construction, on-line barometric sensors to measure the gap pressure, and then the vertical displacement gap.

Fig 12 Vertical loop

[8] Many of the pictures here shown are taken from the book "Matebilandia: Laboratorio di matematica e modellizzazione in un parco di divertimenti" (Resta, Gaudenzi, Alberghi, 2011)

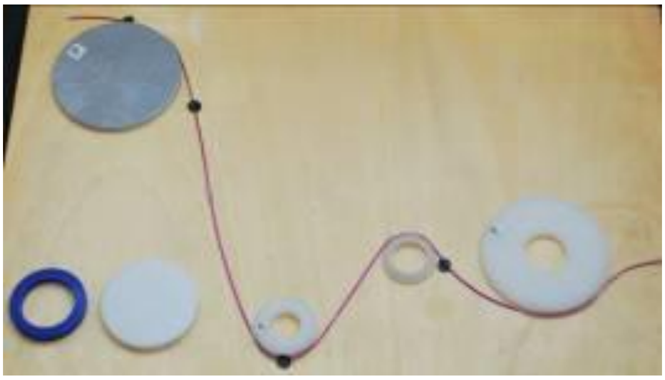

Fig 13Models of curved profiles

One of the most characteristic elements of the roller coaster is the vertical loop. (Fig 12) The top part is shaped like a circumference arc. In order to analyze the part shaped like a "drop", students should be familiar with subjects that are not part of the normal curriculum of Italian high school students: the concepts “curvature” and “osculating circle” are then introduced with the help of examples and models of curved profiles. (Fig 13) Then the students are presented with the clothoid[9], a mathematical curve which faithfully reproduces the characteristics of the curvature of the loop and which finds many practical applications in several fields. A portion of this curve may also be reproduced with an original mathematical machine, called "clothoids-drawer"[10] (Fig 14), whose operation is based on the concepts of osculating circle and evolute of a curve. To sum all up, a "patchwork" of mathematical curves, namely two arcs of clothoid and an arc of circumference, can make up a complete description of the loop. (Fig 15)

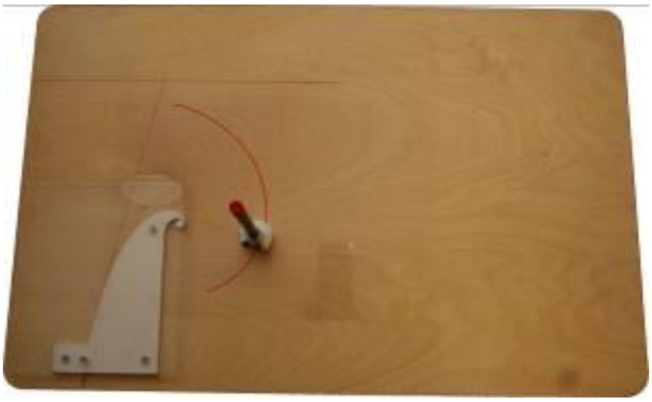

Fig 14 "Clothoids-drawer"

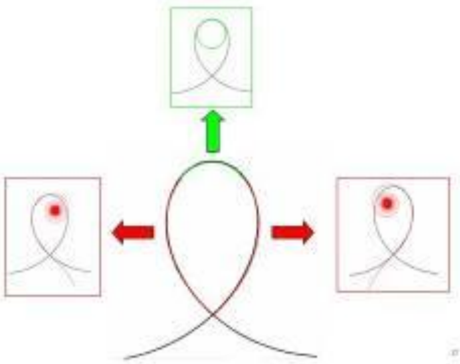

Fig 15 A "patchwork" of curves

[9]This type of curve is used when designing new roads or railways, as in such cases there must be continuity from a straight line and a bent segment. It is also applied in designing building, in art, etc.
[10]The machine was conceived and built by the inventors and developers of the project Matebilandia.

The carousel “Ducks Breakfast” consists of a platform, which rotates at constant speed, on which are placed six cups, each one big enough to array six seats, also equipped with a steering wheel that users can turn freely clockwise or counterclockwise, as they like.

Fig 16 Carousel “Ducks Breakfast”

Fig 17 Epicycloids

Students are asked to predict the shape of the trajectory (epicycloid/hypocycloid) of a “cup passenger” and check this conjecture, using different methods. As first activity a "living model" of the carousel is reproduced, consisting of three people, connected through two ropes, replaying the motion (Fig 18). The combination of the different rotational motions is then sketched in terms of a deferent-epicycle scheme, reminding both the planetary motion Sun / Earth / Moon and the Ptolemaic model. This way students should easily find out the equation of the trajectory parameterized by time.

Fig 18 The "Living model" of the carousel

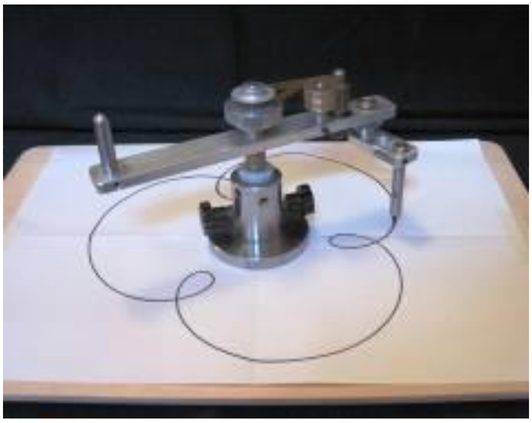

Fig 19 The mechanical model for the carousel

The shape of the rotational movements designed by each "cup" is also verified by means of a miniature model of the carousel, a machine called "Paperografo"[11], (Italian for "Duck graph plotter" built for drawing curves), specially designed for sketching the movements of the carousel.

Fig 20 Students using graphing calculators

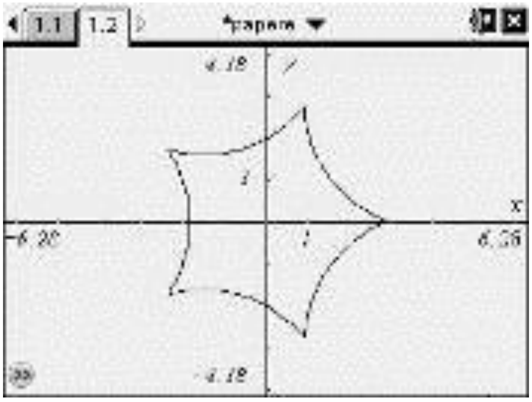

Fig 21 A graph for opposite rotations

[11]The machine was conceived and built by the inventors and developers of the project Matebilandia.

Fig 22 Spirograph

A full exploration of all possible trajectories can also be performed by using suitable software or graphing calculators, and entering the equation of the curve. Such instruments permit to study the opposite rotations of the platform and the cup, or the closing / opening of the trajectory related to the ratio of the angular speeds (Fig 21).

Such movements may also be replicated by using the “Spirograph”, an educational toy based on the rolling of a geared wheel movable along the edge of a geared fixed wheel (Fig 22).

An example of modelling in educational projects

The projects described here below are developed according to a process of modelling and application[12], whose main steps are: understanding, simplification, selection of a mathematical model and practice, interpretation of the results and validation of the model, and final generalization and extension of the model adopted. The process steps are not really always developed according to the sequence stated above, but can be repeated several times, and backward steps may occur.

In this process there are some fundamental elements: the “mathematical discussion" ((Pirie, Schwarzenberger, 1988), (Bartolini Bussi, Boni, Ferri, 1995)), carried out with the group leader and the peers, (both guided and totally free), the group-work and the use of appropriate materials such as concrete objects, measuring instruments, mathematical machines, graphing calculators, worksheets, posters, diagrams and photographs, etc.

Here below we describe as an example the modelling of “Ducks Breakfast”, according to Blum-Leiss cycle (Blum, Leiss, 2005).

– *We start from the real situation and set the targets to be achieved:* we describe the significant elements of the carousel and we single out the shape of the trajectory as our target to study.

– *We simplify the real situation*: observe the overall frame, try to perceive at a glance the shape of the trajectory and clearly detect the two different rotational motions that are combined in the carousel being analyzed.

– *To support understanding, we propose some actual models of the real situation*:

- a "living simulation ", that is a model made up with poor materials and very simply structured. This elementary reconstruction allows to work both theoretically (think of the arrangement of the elements involved), and practically (identifying practical ways to overcome the difficulties encountered in a strictly bound movement).
- The Duck Graph Plotter (thinking of an elaborate model), which reproduces a bare miniature of the carousel, with which you can interact and draw some curves.

– Thanks to the real models and their visible reference to the conceptual framework of epicycle-deferent, you get to the *mathematical modelling* of the trajectory: its equations are parameterized by time and the students’ attention is focused on the quantities on which they depend.

[12]For a comprehensive account on the issue of modelling and applications (M&A) see the ICMI 2007 Meeting report (Blum, et al., 2007).

– *We may comprehensively explore the mathematical model*: you enter the parametric equations of the curves into graphing calculators and analyze the influence of significant parameters on the shape of the trajectory.

– *We may then validate the mathematical model*: we can just juxtapose the various models of the real situation and compare the shape of the trajectories obtained with graphing calculators, with the initial sketch of the carousel, with the living model and the Duck Graph Plotter.

– *We may broaden the view of the problem*: students are introduced to a further method to get the same kind of curves, through the Spirograph, which draws such curves through a combination of different rollings, and they may then discuss of the presence of this type of curves in the surrounding reality.

With regard to the typical steps of a modelling and application process, there are two original elements in the routes proposed: the presence of an unusual environment, specially suited for modelling activities and the use of specific scientific instruments. This project is also purposely and effectively concerned with the students' curricular activities and their further investigation.

What follows is the analysis of each of these aspects.

Specific features of modelling in an amusement park

Much has been said about the learning environment, (for example in (Burkhardt, with the contribution of Pollak, 2006) and (DaPueto, Parenti, 1999)) and also in the educational project Matebilandia this aspect has a central role. As a matter of fact the fair has many positive aspects that recommend its use in teaching:

– It is part of the students' common experience;
– It is a place that arouses curiosity and interest, both in students and teachers;
– It is a place that, of course, brings along the playful;
– It is a meeting place in which the game-playing (or learning, as in this case) are experienced together, in groups, considering the freedom of action and expression that the environment may suggest to the student;
– It is a place that provides many interesting ideas to the scientific disciplines.

Matebilandia tries to seize these opportunities -allowing students to approach learning through fun- and to inspire curiosity about mathematical aspects related to reality. In these activities students are required to express their creativity and to practise different skills, first by working in small groups, then in a group-discussion led by an expert tutor[13].

An out-of-school environment, where to work on problems freely involving real and handling objects, under the guidance of a tutor, does seem useful for an effective and reliable mathematical modelling.

Matebilandia displays and applies such characteristics as there are trained tutors who guide the students through the project, in a "'Classroom Without Walls", away from the schedule, places and methods codified at school. The objects that are being modelled are something real and indeed meaningful to the students, such as the rides of an amusement park: with such things, students can interact not only by looking at them, but also by touching and handling them, or even by experiencing with their whole body. Involving all senses and feelings can make significant and stable the learning (just for instance, better than what can be done in a visual or auditory experience, or at a lecture, movie, and so on[14]).

We want to point out that these " rides" are even "explorable" as real machines in their inner structure. Fair rides are macro-machines very suitable for educational use because their working, built-in technology, and physical and geometrical know-how can be analyzed in details. They are

[13]The tutors, purposely trained by the inventors of this project have the primary task on the one hand to lead the workshop activities towards the expected outcomes, and, on the other hand to elicit all personal contributions from the students, to satisfy their curiosity when dealing with the problems assigned. Regarding the role of the tutor in teaching modelling see (Blomhøj, Kjeldsen, 2006) e (Maaß, 2006).

[14]As to this concern see books dealing with "embodiment", such as, for instance, (Lakoff, Nunez, 2000), or later works dealing with its didactic implications, as in (Arzarello, Pezzi, Robutti, 2007).

engineering products, designed and built on physical and mathematical laws, involving free fall, acceleration, composition of motions, optics, symmetry, and so on.

So fair rides are indeed ready for fun, but they also ready for studying, and they display phenomena less common in everyday life.

In conclusion, we may state that an amusement park is a perfect gym to train for Maths modelling.

The role of mathematical machines

The second remarkable aspect of this project refers to the teaching method we wanted to imprint the modelling process with.

Specific literature on teaching modelling, with features and content extremely varied and interesting, is increasingly being published. Just alike workshop activities, which use real, concrete and handling objects, are gaining an increasingly more significant role in teaching mathematics. See, for example, in Italy the laboratory of Mathematical Machines, University of Modena and Reggio Emilia or the Garden of Archimedes in Florence (Giusti, Conti, 2000). The use of instruments in learning activities in Mathematics and its effectiveness in conveying concepts and meanings is described in (Bartolini Bussi, Maschietto, 2006). References for this type of activity can be searched, for example, in Vigotskij's work (Vigotskij, 1974), with regard to the relationship with the sign, with the language and in general with all cultural artefacts, or in (Rabardel, 1995) for the specific relationship with the *instrument*.

In Matebilandia project, the connection between these two teaching methods, modelling reality and using instruments as in a laboratory, is the core. Since the beginning of the project it was perceived naturally necessary to introduce in teaching modelling, laboratory activities conducted through mathematical instruments and machines: we thought that teaching a modelling process could be effective, challenging and understandable if adequately supported by instruments suitable to provide and/or apply the cycle of modelling and application. Such shifting could more easily have students commute from experiencing in the domain of reality to working in the domain of Mathematics (and vice versa).

Mathematical machines,[15] such as ropes, gears, educational games, and more sophisticated instruments such as graphing calculators or software applications, can lay a bridge between the successive steps of the modelling process. For example, they may provide -or be themselves- the concrete object to experiment on, or on which to validate a mathematical model. They may also suggest analogies, help to better understand a geometric property encountered in their route (taken as an *internal* instrument to mathematics). Mathematical machines may, finally, simply serve as a graphic calculation aid to work on the mathematical model identified. Of course, the same instrument can be relevant in different parts of the modelling process. Similarly a given stage of this process can be performed several times using different instruments.

The aid provided by such machines (some of which designed and built ad hoc by the developers of the project) to the process of modelling is noteworthy, as it enhances the opportunities of manipulation, investigation, exploration, experimentation, play, error correction, validation and refinement of the model . The above description of the process "Ducks Breakfast" and its modelling were meant to exemplify such uses[16].

Such reciprocal relationship probably deserves a more thorough theoretical analysis, aimed in particular at highlighting and understanding the intimate connections between laboratory activities and modelling. Both these aspects have been so far thoroughly analysed, but we believe this was done separately; the relationship between them has not been adequately investigated, and in particular the role of the instruments within the complex cycle of modelling and applications still need deepening. The results of this survey will surely be of great interest, given also the increasing attention that teachers and educational institutions pay to both issues.

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[15]This refers , for instance, to the mathematical machine used in the carousel “Ducks Breakfast”

[16]For further researches see (Resta, Gaudenzi, Alberghi, 2011).

Further possible details of the project

These extra-curricular activities should be bound and arranged in connection with the school programmes, and this would make the experience more effective and learning more stable over time. Materials[17] are available for the teachers to choose from, according to their educational choices, to resume and deepen some of the ideas discussed at the park. In these Maths workshop poor materials, dynamic geometry software, both two- and three-dimensional, graphing calculators, applets, games, have been abundantly used. Suggestions for the teachers and ready-to-use worksheets and files with animations are also provided. The content of these in-depth education projects refer to the conic sections (an analytical and synthetic approach is here displayed and the appropriate references stated), the space analytic geometry, the mathematical machines applied to conics and geometric transformations and an educational game (the Spirograph) concerning topics of Arithmetic and Geometry.

Results

During the first two trials of the project Matebilandia all participating students, after doing the activities at the park, were asked to express their satisfaction anonymously by filling in appropriate forms. Once back to their classrooms, they had also to take standard test to assess their understanding and learning following the activities done at the park.

The satisfaction questionnaires showed that the participants were indeed surprised to discover Mathematics in an amusement park, and this triggered their astonishment and aroused their curiosity to know more, which led to a stronger and more motivated involvement in the activities proposed at the park.

Generally, all of the aspects questioned by the satisfaction form proved warmly appreciated, namely the layout and the content of the handouts prepared to introduce the activities, the effectiveness of the mathematical machines, the use of graphing calculators, the aid of the tutors, the group collaboration.

The understanding tests were mainly open questions, being the goal of the testing to have the students recall the workshop experience in Mirabilandia, to define what they had learnt and explain the method they had followed.

Assessments of comprehension had pretty good marks and were pretty much in accordance with the self-assessments expressed in the questionnaires.

Satisfaction questionnaires were proposed anonymously even to the teachers[18] who took their classes to do Matebilandia projects in the spring of 2009.

Some comments from the teachers showed special appreciation for:

– "The construction of the meaning of mathematical objects, the definitions through conjecture and the verification of information through simple experiments”

– "The opportunity to touch the real issues in which mathematics is applied"

– "The opportunity for the students to get in touch with Maths issues that would otherwise seem distant and abstract"

– The use of mathematical machines and learning through manipulatives;

– The fun that Mathematics can bring along.

Conclusions

The environment chosen and the teaching methodology adopted for our project are the fundamental elements that have led to a constant and active participation of the students, to the desire of discovering Mathematics in the everyday life, reconsidered through "mathematical eyes", and, last but not least, to a good understanding of the content covered.

Nowadays several amusement parks – and even some Carnival fairs- offer rides similar to those analysed at Mirabilandia. We believe that it could be an interesting experience for the teachers and their students, to undergo the (little) burden of organizing an excursion to an amusement park, as we did, and still do.

[17]For further researches see (Resta, Gaudenzi, Alberghi, 2011).
[18]The classes participating came from all over Italy.

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