

Preservice Elementary Teachers' Mathematical Content Knowledge from an Integrated STEM Modelling Activity

Micah S. Stohlmann

University of Nevada, Las Vegas
micah.stohlmann@unlv.edu

Tamara J. Moore

Purdue University
tamara@purdue.edu

Kathleen Cramer

University of Minnesota, Twin Cities
crame013@umn.edu

Abstract

This research was undertaken with one class of preservice elementary teachers to determine the conceptual subject matter content knowledge that was displayed in a well-structured integrated STEM (Science, Technology, Engineering, & Mathematics) modelling activity. Integrated STEM education is gaining more attention for its focus on realistic problem solving and quality pedagogy. The integrated STEM modelling activity used in this research was based on five characteristics of effective STEM curricula: (a) realistic contexts, (b) a focus on math and science content, (c) student-centered pedagogies, (d) technology integration, and (e) the use of the engineering design process. This was a qualitative study that used the Lesh Translation Model as a coding framework for the preservice teachers' written work, audio recordings, and the researcher field notes that were taken. The Lesh Translation Model can be used as a measure of robust content knowledge through five main representations. In this study the preservice elementary teachers demonstrated developed understanding in symbolic, realistic, language, and concrete representations. However, all groups did not demonstrate understanding of the pictorial representation. Research has shown that preservice elementary teachers have room for improvement in mathematical content knowledge. However, when activities are properly structured preservice elementary teachers can demonstrate robust content knowledge.

1 Introduction

Preservice teachers' content knowledge is an essential focus for properly preparing teachers. Yet, no consensus exists on what mathematical content knowledge is needed to teach well (Ball et al., 2001). In the United States research has shown that elementary teachers have room for improvement in robust content knowledge (Ball, Lubienski, & Mewborn, 2001; Ma, 1999). To properly prepare teachers, teacher educators need to demonstrate the importance of well-structured activities. In real life problem solving, skills and processes from many different subjects are often used. Integrated Science, Technology, Engineering, and Mathematics (STEM) education is garnering more attention because of its focus on realistic problem solving and its potential to engage more students in successfully learning mathematics. In order to accomplish this, it is important that preservice teachers have experiences to see how mathematical content knowledge can be used in realistic integrated STEM modelling activities. These activities can be used as authentic assessment tasks that allow participants to display mathematical content knowledge that may not be evident from traditional assessments. The purpose of this paper is to describe the mathematical subject matter content knowledge that one class of preservice elementary teachers demonstrated while working in groups on an integrated STEM modelling activity.

The preservice elementary teachers were enrolled in a mathematics content class that focused on the content of algebraic functions and number and numeration. There were a few activities in the class that integrated mathematics and science and the one activity in this study that integrated STEM concepts. It is important that preservice elementary teachers have robust content knowledge of linear functions and

proportionality to develop algebraic thinking with their students (Blanton & Kaput, 2004). Lewis, Alacaci, O'Brien and Jiang (2002) note that preservice elementary teachers "should be able to use their knowledge of algebraic functions, especially linear functions to model and analyze data, with increasing understanding of what it means for a model to fit data well" (p.178). The integrated STEM activity in this study was used to accomplish this purpose.

1.1 Integrated Mathematics and Science

Since integrated STEM education is relatively new, research in mathematics and science integration can provide insights into teachers' content knowledge for integrated STEM education. There are several studies that have looked at the difficulties of using integrated mathematics and science courses or activities with preservice teachers (Beeth & McNeal, 1999; Frykholm & Glasson, 2005; Furner & Kumar, 2007; Lewis et al., 2002). In an integrated math and science course Beeth & McNeal (1999) found that the preservice teachers felt that in mathematics the main question tends to be, what is the answer? While in science it tends to be, why? In another study, Frykholm & Glasson (2005) found that preservice teachers, based on their own experience, felt that mathematics was typically fragmented and taught in isolation of other topics. In developing an integrated mathematics and science curriculum project the preservice teachers began to become concerned about their own integrated content knowledge since they realized their knowledge for this instruction was lacking. However, the preservice teachers felt that integrated education was important and possible.

A study with preservice elementary teachers enrolled in a science course provides more information on mathematics content knowledge. The preservice teachers developed a project that was supposed to integrate mathematics and science. However, the preservice teachers underused mathematical representations. When it would have been appropriate the preservice teachers did not use scatterplots or model the data with linear functions. The mathematics classes that the preservice teachers had taken focused mainly on symbolic work with little practice in applications (Lewis et al., 2002).

Continued emphasis on improving teacher content knowledge is a prerequisite to enabling teachers to integrate content. Teacher educators need to help preservice teachers develop mathematics content knowledge while simultaneously introducing them to new concepts and ideas from science disciplines (Stinson, Harkness, Meyer, & Stallworth, 2009). Stinson et al. (2009) noted that a common understanding for what it means to integrate mathematics and science appears to be a significant obstacle to an integrated approach to instruction. For integrated STEM education a common understanding is still being developed as well.

1.2 Integrated STEM Modeling Theoretical Framework

There are five main characteristics of effective integrated STEM curricula. First, the context for the curricula should be motivating, meaningful, and engaging for the students (Brophy, Klein, Portsmore, & Rogers, 2008; Frykholm & Glasson, 2005). Second, there must be mathematics and/or science content as the main objectives of the activity (Stohlmann, Moore, McClelland, & Roehrig, 2011). Third, student-centered pedagogies should be used in the activities including teacher as a facilitator and cooperative learning (Johnson, Johnson, & Smith, 2007). Fourth, the activity should enable students to become more technology savvy through the use or design of technology or through understanding how technology has developed over time. Finally, the activity should have students participate in the engineering design process that enables them to develop teamwork and communication skills; as well as the ability to work with a diverse group of people (Dym, Acogino, Eris, Frey, & Leifer, 2005).

There are two core ways to implement integrated STEM education: content or context integration. Content integration focuses on the merging of the content fields in order to highlight "big ideas" from multiple content areas. For example, students could be asked to recommend the top five most promising and sustainable energy sources for a country to focus their energy investments in for the future. In this modelling problem students would use science content knowledge of non-renewable and renewable resources. Categorical data could be given to students that rates energy sources on a 5 point scale on

categories such as easily transported, low greenhouse gases, and easily used for transportation. For the mathematics content, students could use statistics concepts of measures of center or correlation to help make their decision. The other method for integrated STEM education, context integration, primarily focuses on the content of one discipline and uses contexts from others to make the content more relevant. For example, students could be given a footprint that might have belonged to the legendary creature Bigfoot. They could be then asked to come up with an estimate for the height of the person or creature based on the footprint. While science concepts of observation and inference are included in this activity, what is being asked of students does not require the explicit use of science content to develop a solution.

STEM integration should only be used when there are natural connections between subjects. This enables this approach to motivate and engage students in meaningful learning (Furner & Kumar, 2007). If integration is forced the activities will seem contrived, unrealistic, and disconnected. For this paper integrated STEM education will be defined as an approach for teachers to use the engineering design process as the structure for students to learn mathematical content in a science context through technology infused activities. An effective structure for accomplishing this is through the use of Model-Eliciting Activities (MEAs).

MEAs are client-driven, open-ended, realistic problems that involve the development or design of mathematical/scientific/engineering models. MEAs are not meant to be a full curriculum but to complement the content of a course. The goal of MEAs is to have students develop models or solutions that are powerful, sharable, and reusable (Lesh, 2010). The engineering design process is built into these activities as students express, test, and revise their solutions. Many MEAs are set in science contexts where students use mathematical ideas to make sense of the tasks given by a realistic client. These activities are broadening classroom experiences that tap the diversity of learning styles and strengths that students bring to the classroom.

MEAs can be difficult to write and are developed based on six design principles (Lesh, Hoover, Hole, Kelly, & Post, 2000). These principles (Table 1) help to ensure that MEAs provide students the kind of rich learning environment that allows for students to develop good problem solving skills and allow instructors and researchers to “see” the thinking of students.

Table 1.
Principles for Guiding MEA Development

Principle	Description
<i>Model Construction</i>	Ensures the activity requires the construction of an explicit description, explanation, or procedure for a mathematically significant situation.
<i>Generalizability</i>	Requires students to produce solutions that are shareable with others and modifiable for other closely related situations.
<i>Model Documentation</i>	Ensures that the students are required to create some form of documentation that will reveal explicitly how they are thinking about the problem situation.
<i>Reality</i>	Requires the activity to be posed in a realistic context and to be designed so that the students can interpret the activity meaningfully from their different levels of mathematical ability and general knowledge.
<i>Self-Assessment</i>	Ensures that the activity contains criteria the students can identify and use to test and revise their current ways of thinking.
<i>Effective Prototype</i>	Ensures that the model produced will be as simple as possible, yet still mathematically significant for learning purposes (i.e., a learning prototype, or a “big idea” in mathematics).

1.3 Conceptual Subject Matter Knowledge

MEAs are often structured for students to work with multiple representations and use subject matter knowledge and skills from different subjects. The Lesh Translation Model (Figure 1) is a measure of robust content knowledge through five main representations and translations between and within these

representations: (a) Representation through realistic, real-world, or experienced contexts, (b) Symbolic representation, (c) Language representation, (d) Pictorial representation, and (e) Representation with manipulatives (concrete, hands-on models). The translation model emphasizes that the understanding of concepts lies in the ability of students to represent concepts through the five different categories of representation, as well as the ability to translate between and within representations (Lesh & Doerr, 2003). As participants work on MEAs they can show misconceptions and partial understanding but by the end of the activity groups often progress to more well developed understanding as shown through multiple representations (Lesh & Doerr, 2003). The Lesh Translation Model is used as the measure of mathematical content knowledge in this study.

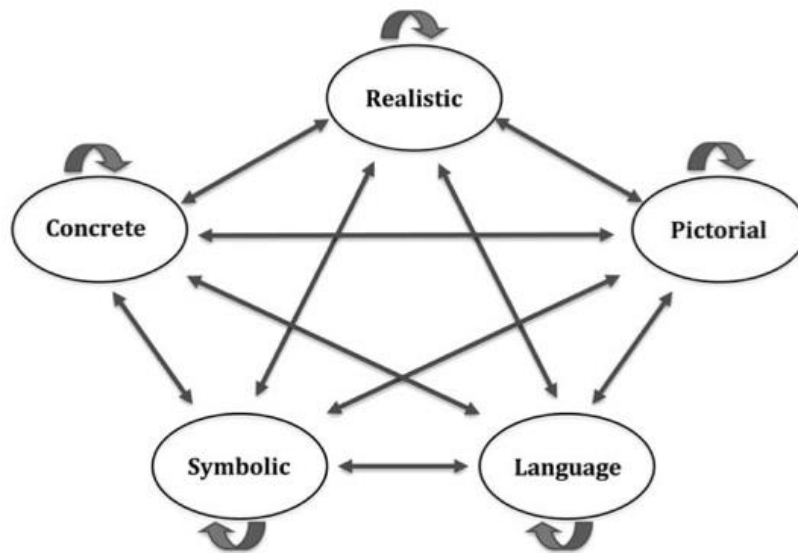


Figure 1. The Lesh Translation Model

2 Methods

2.1 Setting

The setting for this study is a large Midwestern public university in the United States. The preservice elementary teachers in this study were enrolled in the first of two required mathematics content classes. The preservice teachers were also required to complete a mathematics methods class. The content class in this study met twice a week for eighty minutes. The class focused on the content of algebraic functions and number and numeration. The preservice teachers completed the MEA at the conclusion of the functions unit. The functions and proportionality portion of the class is organized around the exploration of linear, quadratic, and exponential functions through multiple representations. The preservice teachers then use their knowledge of functions, especially linear functions to understand the characteristics of proportional situations.

2.2 Participants

One class of thirty preservice elementary teachers, twenty-six women and four men, comprised the sample. The preservice elementary teachers were selected based on their enrollment in the mathematics and pedagogy class. The instructor for the class was an associate professor in mathematics education.

2.3 Bigfoot MEA

The Big Foot MEA has been used in previous research to investigate students' reasoning with concepts related to proportionality (Lesh & Doerr, 2003; Lesh & Harel, 2003). The authors modified the context of the Big Foot MEA, while keeping the general structure of the problem. Table 2 contains the problem statement that the preservice teachers were given. The realistic client for this MEA is the Northern Minnesota Bigfoot Society whose mission is to collect data that might lead to the proof of the legendary Bigfoot creature.

Table 2.
Bigfoot MEA Problem Statement

<p>Problem Statement</p> <p>The Northern Minnesota Bigfoot Society would like your help to make a “HOW TO” TOOLKIT; a step-by-step procedure, they can use to figure out how big people are by looking at their footprints. Your toolkit should work for footprints like the one that is shown on the next page, but it also should work for other footprints.</p>

The format of this activity followed a progression used in most MEAs. The preservice teachers individually read an opening article that described information from their client and about how a tracker uses footprints. The preservice teachers then answered three questions related to the readings and discussed their answers in a whole class discussion. Next, a short video was shown to the preservice teachers to illustrate how a concept as simple as a footprint has many different integrated connections. The video shows how forensic scientists, professional trackers, and scientists use footprints as evidence in various ways to make conclusions (FlowMathematics, 2011). The preservice teachers then worked in their groups to answer the problem. Materials were available for the groups to use including rulers, meter sticks, string, scissors, graph paper, graphing calculators, and laptops. After groups developed their solutions, each group shared their ideas and then time was given for groups to revise their ideas. Finally, a short overview of MEAs was discussed with the preservice teachers along with the integrated STEM connections that were incorporated in the activity.

Table 3 describes the STEM connections in this MEA. Science process skills are evident in this MEA through the whole class discussion at the beginning of the MEA of how scientific evidence is used and how science knowledge can change over time. Technology resources were available for the preservice teachers if they chose to use them. The activity has the preservice teachers work through a modified form of the engineers design process and there are many possible mathematical concepts that the preservice teachers may use in this activity.

Table 3.
STEM Connections in the Bigfoot MEA

MEA	Science	Technology	Engineering	Mathematics
Bigfoot	Observation and inference Measurement Science as a way of knowing	Graphing calculators or computers	Design process	Ratios Proportions Line of best fit Scatter plots Plotting points Estimation Measurement Sampling methods Outliers Sample Size Measures of center

2.4 Data collection

In order to see how groups progressed to their final solution, audio recordings of each group were collected as well as the groups' final solutions and their written work throughout the problem. In addition, one of the researchers took field notes while groups were working, focused on any representations that groups demonstrated while working through the problem.

2.5 Data analysis

The Lesh Translation Model (Figure 1) was used as a coding framework for the mathematical subject matter knowledge that the preservice teachers demonstrated. Each data source was analyzed first by coding for evidence of the five representations: realistic, symbolic, concrete, language, and pictorial (Table 4). Next, each data source was analyzed and compared by looking for any translations between representations.

Table 4.
Bigfoot MEA Content Knowledge Framework

Representation	Framework
Realistic	Involved body parts, Bigfoot, measurements and background knowledge
Pictorial	Scatterplot or graph
Language	Written or verbal communication connected to the concepts of linear equations, proportionality, or statistics
Symbolic	Measurements, averages, ratios, linear regression equation, algebraic manipulation, and conversions
Concrete	Body parts and Bigfoot's footprint

3 Results

The Bigfoot MEA was completed by the preservice teachers in groups of five or four on the last day of the functions unit. Overall, the preservice teachers demonstrated conceptual understanding of linear functions in the Bigfoot MEA through translations within and between representations with only the pictorial representation not demonstrated by all groups. Pictorial representations were used by three of the seven groups and translations within pictorial representations were not evident. These findings will be illustrated below. First each group's solution will be discussed. Then the iterative process that occurs in MEAs will be highlighted with one group's model development. Finally the classes' use of the five representations will be discussed and the groups' translations between and within representations will be presented.

A summary of each group's final solution is presented in Table 5. Groups had initial ideas that eventually went away through discussion or were refined. These ideas included looking at the ratio of feet length to feet width, using weight, using Bigfoot's foot width, using a linear regression equation with height as the independent variable and foot length as the dependent variable, using a linear regression equation with height and foot lengths that did not have much variation, and using wingspan related to foot length.

Table 5.
Groups' Final Ideas on the Bigfoot MEA

Group	Final idea
1	Calculated ratios of height to foot length for each group member and then averaged the ratios. Then multiplied this by Bigfoot's foot length.
2	Calculated the ratios of height to foot length for each group member, estimated that the ratios were all close to 6, and then multiplied 6 by Bigfoot's foot length.
3	Used the average human foot length to height ratio of 15 to 100 found on an internet website to set up a proportion with Bigfoot's foot length. Noted that a larger sample size is needed to be able to use a regression line.
4	Used the average human foot length to height ratio of 15 to 100 found on an internet website to

	set up a proportion with Bigfoot's foot length. They also liked the idea to take the average of each group member's foot length and divided it by the average of each group member's height which gave them a ratio of 14 to 100, which they could use similarly to set up a proportion with Bigfoot's foot length.
5	Calculated ratios of height to foot length for each group member, averaged the ratios, and then multiplied this by Bigfoot's foot length.
6	Used each group member's foot length and height and entered this into the graphing calculator to find a linear regression equation; then plugged in Bigfoot's foot length into the equation.
7	Used all but one group member's foot length and height and entered this into the graphing calculator to find a linear regression equation. One group member's data was not used because the group felt the data looked closer to being linear without it. They then plugged in Bigfoot's foot length into the equation. They noted that ideally you would want to get height and foot lengths of a sample of a couple hundred people.

Group 3's progression of ideas will be shown to illustrate the development of solutions that occurred in the groups. For the representative parts of the group's work that will be used, the representations that were evident will be included after each part in parentheses. Initially, the group members discussed different ideas related to developing a solution.

S1: "I think a solution might be if there is a correlation between your foot and the size of your body. I am thinking that if we measure our feet and put it relative to our height were going to find that correlation." (Language and Realistic)

S2: "Should we do width as well?" (Language and Realistic)

S3: "I don't know if we have the sample size to do this." (Language)

The group decides to measure Bigfoot's footprint length and measures their feet length. Then they share this and their height. (Concrete and Realistic) The group looks at how the x and y data are changing and talks about not using one data point.

HEIGHT (y)	FOOT (x)
68 in	$9\frac{3}{4}$ in
63 in	10 in
66 in	9 in
69 in	10

S3: "If we throw out yours (63 inch height and 10 inch feet length) that would be a line. You are such an outlier. Remember that word you are more than two standard deviations away from the mean." (Language, Realistic, and Symbolic)

The group does not check if the point is an outlier based on the definition that one group member mentioned. They decide to enter all four data points into the graphing calculator and use the linear regression feature to fit an equation.

S1: "So, I put it into my calculator and got $y = .186x + 64.7$ " (Symbolic)

One of the students looks at the scatterplot on the graphing calculator to see if a linear function is appropriate and noticed the trend that was seen in the table.

S4: "Well these three points are linear." (Language and Pictorial)

The group then decides to plug in Bigfoot's foot length into the equation to see what the estimate would be. (Symbolic, Language, and Realistic)

$$y = .186 (15\frac{1}{2}) + 64.7$$

67 inches = big foot

This height seems too short though.

S2: *"It doesn't really make sense. That would be similar to our heights"* (Language and Realistic)

S3: *"That is the thing we do not have a big enough sample size."* (Language)

Another student looks at the data in the table to make an observation on the lack of variance of the heights and feet lengths.

S1: *"And look how close we are all together. We could use someone else's data."* (Language, Realistic, and Symbolic)

The group gets height and foot length measurements from another group.

ADDITIONAL

62	9.5 in
63	10.5
65.5	10.5
69.5	10.75

(Realistic and Symbolic)

The group uses all eight data points now to get a regression equation.

$y = -.578x + 70.9$ (Symbolic)

S3: *"So, again we need a bigger sample size."* (Language)

S4: *"I think it is because they are so close together. It is almost a straight line across."*

(Language and Pictorial)

The time was approaching when each group had to share their ideas so the group decides to use one group member's height and foot length to set up a proportion with Bigfoot's foot length to find an estimate of Bigfoot's height. (Language, Realistic, and Symbolic)

After hearing other groups share their idea, the group decides to use an idea from another group to set up a proportion using 15 over 100 and Bigfoot's foot length.

S1: *"Human foot is 15% of height."* (Language, Realistic, and Symbolic)

In total the group demonstrated conceptual understanding in different representations and were able to develop a solution to meet the needs of the client. When the group used all eight data points they looked at the scatterplot and saw that it was not linear besides giving an estimate of Bigfoot's height that seemed too low to be realistic. In the end, they decided on a solution that used a much larger sample size to see on average what the ratio of foot length to height is for humans. The following is a summary of representations that were evident from all groups on the Bigfoot MEA and then the translations between and within representations will be discussed.

Pictorial.

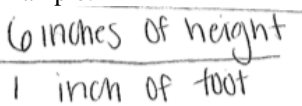
Pictorial representations were used by three groups and involved using a scatterplot on the graphing calculator to decide if a relationship was somewhat linear.

Realistic.

There were a variety of realistic representations including various body parts and real world situations related to feet including ballerina's feet, clown shoes, and Shaquille O'Neal's shoes. Table 6 has a summary of the realistic representations and how many groups they appeared in. Throughout the activity the conversations involved the context of the problem and the best way to solve the problem based on prior knowledge and experiences.

Table 6.
Number of Groups with Each Realistic Representation

Realistic Representation	Number of Groups
Each group member shared his or her heights.	7
Found the length of Bigfoot's footprint.	7
Discussed their solution, the height of Bigfoot.	7

Each group member shared his or her foot length.	7
Suggested looking at the ratio of height to foot length.	6
Described conversions from inches to feet for height.	6
Discussed how to measure their feet (shoes on or off and where to measure).	5
Shoe size was suggested to be used, but then was not used.	3
Described the ratio of height to foot size. Example: 	3
Suggested using Google to generate ideas.	3
Described how to use Bigfoot's foot length with an equation or a ratio.	3
People vary. It is not always the case that if you are taller you have bigger feet.	2
Discussed how to measure Bigfoot's footprint.	2
Described how to set up a proportion with average foot length, average height, and Bigfoot's foot length.	2
Described how to set up a proportion with one person's height and foot length.	2
Suggested to look at the width of group members' feet or Bigfoot's footprint	2
"If we throw out yours (63 inch height and 10 inch feet length) that would be a line. You are such an outlier."	1
Suggested finding the average human ratio of height to foot length.	1
Each group member shared his or her hand length.	1
One group member shared their forearm length and shoulder length.	1
Discussed how foot length relates to forearm length and upper arm length.	1
One group member described personal experience of knowing that wingspan is not always the same as height.	1
Stated that their solution method only works for humans.	1
Wanted to combine average human height to foot ratio with the average gorilla height to foot ratio.	1
Foot length is 15% of height on average.	1
Scatterplot has to be linear because feet cannot grow exponentially.	1
Suggested using a stride length or weight to help find a solution.	1
Some measurements could have been more precise.	1
A better estimate would be possible if a sample of a couple hundred peoples' height and foot lengths could be taken.	1
Described why the line of best fit gave a bad or unrealistic estimate. The group's heights and foot lengths were too similar.	1

Language.

The language representations occurred throughout the activity and overlap with the realistic representations (Table 7). Many of the realistic representations in Table 6 above are also language representations as well. Just the language representations that are not also realistic representations will be summarized in this section.

Table 7.
Number of Groups with Each Language Representation

Language Representation	Number of Groups
The sample size is too small to do this. Example: "So, again we need a bigger sample size."	2
Described an outlier as "more than 2 standard deviations away."	1
Graph should be linear. "It is going to be linear too." "That is a good guess lets put that down. It might be linear."	1
Our solution makes sense because we found the scale factor.	1

Symbolic.

The symbolic representations occurred when groups wrote out different measurements, calculated averages, calculated ratios, wrote a linear regression equation, did algebraic manipulation, and converted a foot length written as a decimal to inches. Table 8 has a summary of the number of groups that used each symbolic representation.

Table 8.
Number of Groups with each Symbolic Representation

Symbolic Representation	Number of Groups
Made a table with height and foot lengths	7
Wrote out Bigfoot’s foot length	7
Plugged Bigfoot’s foot length into an equation, proportion, or multiplied it by a number	7
Wrote a linear regression equation from the graphing calculator	3
Plugged a group member’s foot length into a linear regression equation to see if the prediction is close to the actual height	2
Wrote a linear regression equation with height as the independent variable and foot length as the dependent variable	2
Calculated the average foot lengths and heights of group members	2
Calculated ratios of height to foot length for each group member	2
Wrote each group member’s foot widths	1
Calculated the ratio of foot length to width	1
Averaged the ratios of height over foot length	1
Found an estimate of ratios	1
Converted a foot length as a decimal to inches	1

Concrete.

There were different concrete representations that were used by groups in this activity that were essential to developing a solution for this problem. Every group used the footprint of Bigfoot, used their own body parts, and used a meter, ruler, or string to help them collect measurements. Table 9 has a summary of the concrete representations.

Table 9.
Number of groups with each concrete representation

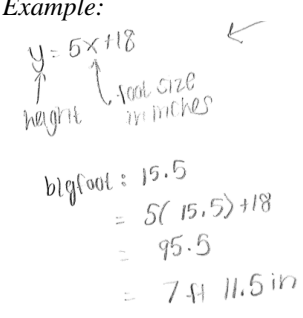
Concrete Representation	Number of Groups
Used a ruler, meter stick, or string to measure	7
Measured Bigfoot’s footprint	7
Measured group members’ body parts	7

Translations between representations.

Throughout the activity groups demonstrated the ability to translate between representations in a number of ways. Every group translated between the representations of concrete, realistic, language, and symbolic in different orders and combinations. Three of the groups translated between pictorial representations to concrete, language, and symbolic representations. Table 10 has a summary of the translations between representations that occurred.

Table 10.
Translations Between Representations and Examples

Translation	Description and Example(s)
LR (7 groups)	This translation involved discussing how to measure body parts, describing Bigfoot’s footprint, and other real world situations related to feet. <i>Example:</i> “We could look a ratio of height to feet length.”

	<i>Example:</i> "This will only work if Bigfoot is human."
LRS (7 groups)	<p>This translation involved creating a table of measurements with units, symbolic manipulation with units or description, sharing body measurements with units, or explaining a solution process.</p> <p><i>Example:</i> "What we ended up doing is we divided our height by the length of our feet and found them to be around 6. So we just took the bigfoot times 6."</p> <p><i>Example:</i></p> 
CLRS (7 groups)	<p>This translation involved using body parts or Bigfoot's foot to make measurements and then describing the measurements with units.</p> <p><i>Example:</i> "You can do your foot times 4 and this is your footprint and a half or 1/3 is what we said, but then you also need to figure out how big your hands are compared to your feet, because your hands also play a role because it is fingertip to fingertip."</p> <p><i>Example:</i> (after measuring feet) "My foot is 10 inches long."</p>
LS (7 groups)	<p>This translation involved a description without a context of symbolic work.</p> <p><i>Example:</i> Linear regression line: $y = -.578x + 70.9$</p> <p><i>Example:</i> "If we throw out yours, that would be a line then. 63 and 10. You are such an outlier."</p>
CLPRS (3 groups)	<p>This translation occurred when groups described what they had done to solve the problem.</p> <p><i>Example:</i> "We first found our own data and measured the height with the foot size and then made the foot size our x and the height our y and we found the linear regression line and found it was not accurate. So we tried to get more data to enlarge our sample size, which still really did not help. So if we were to use our linear regression line he would have been about 6 feet tall or 61 inches which is not at all feasible so then I set up a proportion to my own height and foot size, because I am 5 foot 6 and have 8 inch feet which is pretty average. So using that I found that he was 9 feet and 7 inches, but I like the other one (solution idea) better with the 15 to 100."</p>

Note: C=Concrete, P=Pictorial, L=Language, R=Realistic, S=Symbolic

Translations within representations.

Translations within representations occurred in the symbolic, realistic, concrete, and language representations for all groups. The translations within symbolic representations occurred between a table of measurements, equations, ratios, plugging in for x and solving for y in an equation, conversions of units, and calculating averages. The translations within realistic representations occurred between measuring body parts, measuring Bigfoot's footprint, and discussing shoes. The translations within concrete representations occurred between different body parts and the Bigfoot footprint. The translations within language representations occurred between discussing graphs, giving measurements, describing possible ideas, assessing the feasibility of a solution, and generating alternative solutions.

In summary, all of the groups were able to progress through the activity and work towards a solution that met the needs of the client. Through discussion the groups were able to move away from unproductive ideas and towards an understanding of how to come up with an estimate for the height of Bigfoot. All groups demonstrated the ability to show conceptual understanding of linear functions through different representations and translations between and within representations. Figure 2 has a graphical summary of the conceptual understanding from the Bigfoot MEA. The dotted lines going to and around

the pictorial representation shows that only three groups showed understanding in this representation. The solid lines show fully developed conceptual understanding. The arrows going back into each representation denote the translations within representations demonstrated. Fully developed conceptual understanding would have solid lines throughout like the Lesh Translation Model (Figure 1).

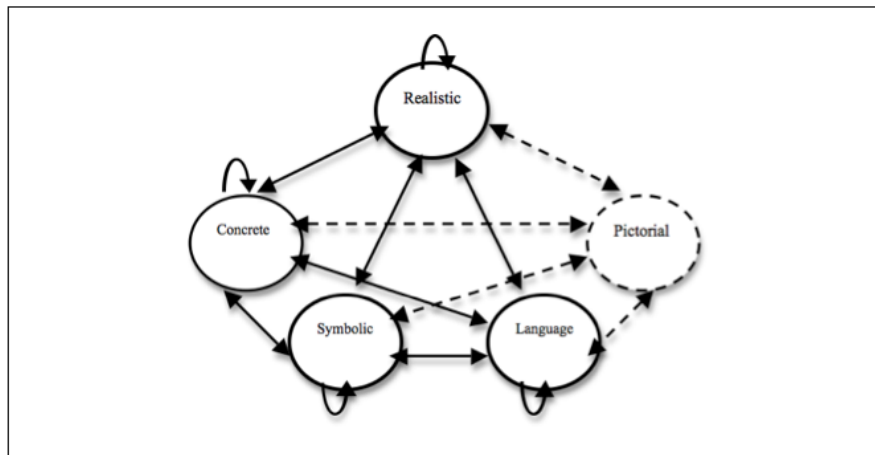


Figure 2. Preservice Teachers' Conceptual Understanding from Bigfoot MEA

4 Discussion

The preservice teachers in this study demonstrated conceptual content knowledge on an integrated STEM modeling activity. They were able to show understanding of proportionality and linear function modelling through different representations and translations between and within representations. In Lewis et al. (2002) study that focused on preservice elementary teachers applying their knowledge of linear functions on an integrated science and mathematics project the preservice teachers underused mathematical representations. However, since the preservice teachers in this study had not had learned mathematics with a focus on representations, this is not surprising. Many preservice elementary teachers, based on their own schooling experience, believe that mathematics mainly involves symbolic work. It takes new experiences for preservice teachers to see the benefits of learning mathematics through different representations. Similarly to Lewis et al. (2002) less than half of the preservice teachers used scatterplots or linear regression. The graphical representation was the one representation lacking from the preservice teachers' work. Realistic and language representations were the most common, with different calculations being done of body parts in the context of finding Bigfoot's height.

Previous research in mathematics and science integration has shown that preservice teachers view mathematics as an isolated topic (Frykholm & Glasson, 2005) and that mathematics is focused on getting the right answer while science can be more open-ended (Beeth & McNeal, 1999). It is important that preservice elementary teachers are shown how mathematics can be open-ended which more closely mirrors real-life problem solving. There were a variety of solutions that were developed including the use of linear regression equations, ratios, averages, and sample size considerations.

There are five main characteristics of effective integrated STEM curricula that were incorporated in the Bigfoot MEA.

- First, the context of the activity was motivating and engaging for the preservice teachers (Brophy et al., 2008; Frykholm & Glasson, 2005). Based on the researcher field notes and the audio recordings, the preservice teachers' conversations were intently focused on developing a solution to the problem. The conversations also involved the preservice teachers making connections to their own lives with comments related to body parts that are related, feet size, and knowledge of Bigfoot.
- Second, mathematics content and science concepts were the main objectives of the activity (Stohlmann et al., 2011). The preservice teachers were able to use their knowledge of ratios,

proportions, measurement, linear function modelling, and ideas related to sample size to develop their solutions. MEAs are a useful structure for integrated STEM activities that are organized around mathematical big ideas. Since MEAs integrate different representations, preservice teachers are able to demonstrate conceptual content knowledge. In the Bigfoot MEA science concepts were integrated through the opening article and YouTube video on integrated connections to footprints. The preservice teachers were able to discuss how people can observe the same thing but have different inferences for what their senses have observed.

- Third, student-centered pedagogies were used in the activity. The instructor structured the class activity so that the preservice teachers could pose problems about natural phenomena, probe for answers that explain a problem, and persuade their peers their solution is sufficient (Beeth & McNeal, 1999). The instructor was a facilitator as groups worked together and explained their ideas to the whole class (Zemelman, Daniels, & Hyde, 2005). The activity also allowed the participants to build on their prior knowledge (Berlin & White, 1995).
- Fourth, the activity integrated a narrow view of technology with electronic technology in the form of graphing calculators. In a general sense, technology is anything that is human made that makes life easier. With this view, measurement devices were also a form of technology integrated into this activity. While it was not included in this activity, the development of measurement devices over time could also have been discussed. The preservice teachers discussed how various body parts were related and that not everyone's body is the same. Since ancient measurements were based on body parts, it shows why the metric system and the customary system were developed.
- Finally, the activity was structured with a modified form of the engineering design process and engineering thinking. As the preservice teachers worked through the activity they expressed, tested, and revised their ideas. They moved away from initial ideas such as using weight, wingspan, the ratio of feet length to feet width, using Bigfoot's foot width, and using a linear regression equation with height as the independent variable and foot length as the dependent variable. The information provided in the activity, the other group members, and other groups in the class allowed for the students to self-assess their own solutions and to develop a solution that met the needs of their client. The activity had no one single right answer so the preservice teachers attempted to develop the best solution that they could.

Teacher content knowledge and classroom practices are important considerations for integrated education (Stinson et al., 2009). If teachers do not have a solid understanding of what constitutes effective STEM integration then it may be implemented poorly. For teachers new to integrated STEM education, content knowledge and teaching practices are the most important considerations (Stohlmann, Moore, & Roehrig, 2012). Integrated STEM education enables learning to be connected and more realistic to the type of problem solving that is done in everyday life. Future research can focus on elementary grade level MEAs used with preservice teachers in methods courses; as well as the impact of sharing research-based models of elementary students' thinking on MEAs with preservice and practicing teachers.

References

- Ball, D. L., Lubienski, S. T., & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook on research in teaching (4th edition)*, pp. 180-194. New York, NY: Macmillan.
- Beeth, M., & McNeal, B. (1999). Co-teaching science and mathematics methods courses. Retrieved from ERIC database. (ED443666).
- Berlin, D., & White, A. (1995). Connecting school science and mathematics. In P. House & A. Coxford (Eds.), *Connecting mathematics across the curriculum: 1995 yearbook* (pp. 22-33). Reston, VA: National Council of Teachers of Mathematics.

- Blanton, M. & Kaput, J. (2004). Elementary grades students' capacity for functional thinking. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, 2, 135-142.
- Brophy, S., Klein, S., Portsmouth, M., & Rogers, C. (2008). Advancing engineering education in P-12 classrooms. *Journal of Engineering Education*, 97, 269-387.
- Dym, C., Acogino, A., Eris, O., Frey, D., & Leifer, L. (2005). Engineering design thinking, teaching, and learning. *Journal of Engineering Education*, 94(1), 103-120.
- FlowMathematics (2011). Footprint facets. Retrieved from <http://www.youtube.com/watch?v=2SIUdSwYuv0>
- Frykholm, J., & Glasson G. (2005). Connecting science and mathematics instruction: pedagogical context knowledge for teachers. *School Science and Mathematics* 105(3), 127-141.
- Furner, J., & Kumar, D. (2007). The mathematics and science integration argument: a stand for teacher education. *Eurasia Journal of Mathematics, Science & Technology*, 3(3), 185-189.
- Johnson, D.W., Johnson R.T., & Smith, K. (2007). The state of cooperative learning in postsecondary and professional settings. *Educational Psychology Review*, 19, 15-29.
- Lesh, R. (2010). Tools, researchable issues & conjectures for investigating what it means to understand statistics (or other topics) meaningfully. *Journal of Mathematical Modeling and Application*, 1(2), 16-48.
- Lesh, R. & Doerr, H.M. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & H.M. Doerr (Eds.), *Beyond constructivism*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Lesh, R. & Harel, G. (2003). Problem solving, modeling, and local conceptual development. *Mathematical Thinking and Learning*, 5(2&3), 157-189.
- Lesh, R., Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). *Principles for developing thought-revealing activities for students and teachers*. In A. Kelly, R. Lesh (Eds.), *Research Design in Mathematics and Science Education*. (pp. 591-646). Mahwah, New Jersey: Lawrence Erlbaum Associates.
- Lewis, S., Alacaci, C., O'Brien, G., & Jiang, Z. (2002). Preservice elementary teachers' use of mathematics in a project-based science approach. *School Science and Mathematics*, 102(4), 172-180.
- Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Stinson, K., Harkness, S., Meyer, H., & Stallworth, J. (2009). Mathematics and science integration: models and characterizations. *School Science and Mathematics*, 109(3), 153-161.
- Stohlmann, M., Moore, T., & Roehrig, G. (2012). Considerations for teaching integrated STEM education. *Journal of Pre-College Engineering Education*, 2(1), 28-34.
- Stohlmann, M., Moore, T., McClelland, J., & Roehrig, G. (2011). Year-long impressions of a middle school STEM integration program. *Middle School Journal*, 43(1), 32-40.
- Zemelman, S., Daniels, H., & Hyde, A. (2005). *Best practice: New standards for teaching and learning in America's school* (3rd Edition). Portsmouth, NH: Heinemann.