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Studies on the Mathematical Expertise of Mechanical Engineers

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Abstract
This contribution summarises the results of a four-year project that aimed at capturing the mathematical expertise a rather practice-oriented mechanical engineer needs in his or her daily work. After a comprehensive review of the literature on engineering workplace studies regarding mathematics, we present and justify the method of investigation chosen for the studies. Eight students worked on four different "typical" tasks for mechanical engineers that were specified in cooperation with a colleague in engineering. Based on a detailed investigation of their documentations, tool usage and interviews with students and the colleague we identify components of mathematical expertise needed for working on the tasks and compare them with literature results. The findings corroborate many results from studies on civil and industrial engineering and add some new or more detailed results in particular regarding the effective and efficient use of computational tools and qualitative and quantitative models in mechanical engineering tasks.

Keywords: workplace studies; mathematical expertise; mechanical engineering

1. Introduction

German universities of Applied Science offer a rather practise-oriented education. Correspondingly, the mathematical education of engineers has two major goals: It should enable students to understand, set up and use the mathematical concepts, models and procedures that are used in the application subjects like engineering mechanics, machine dynamics or control theory. In order to clarify this first goal, a lecturer has to investigate the use of mathematics in application textbooks or scripts of colleagues. Although this is also demanding and time-consuming for a non-engineer, the information is quite readily available. The second major goal of mathematics education is to provide students with a sound mathematical basis for their future professional life. For clarifying this rather nebulous goal, one needs information on the mathematical expertise or qualifications an engineer needs in his or her daily life. This information is not easy to acquire, and because there are so many different branches of engineering and job profiles within these branches, it is appropriate to restrict oneself to a certain job profile within a specific branch. This contribution aims at shedding more light on the mathematical expertise a practice-oriented mechanical engineer (a graduate of our kind of educational institution) needs.

In the last two decades, several studies on mathematics at the workplace have been conducted (Bessot and Ridgway 2000, Gainsburg 2005). Most of these studies dealt with rather "lightweight" use of school mathematics. Just a few studies captured presumably heavy users of mathematics like engineers and scientists. In the first section of this contribution, we review these studies. We then present and justify our own method of study. Eight students in their final semester worked on four "typical" tasks we specified in cooperation with a colleague from engineering. Section 4 gives a short overview of these tasks. We then present our findings and relate them to literature results in section 5.[1]

2. Literature Review

Among the studies on engineering workplace mathematics, those by Kent and Noss (2002, 2003) and by Gainsburg (2006, 2007a,b) deal with structural engineers and those by Hall (1999) and Steevens and Hall (1998) refer to architects. These branches are nearer to everyday life and hence the

[1] The results of our single task investigations have been published in more detail in conference proceedings and project reports (see the references). This article gives a comprehensive and summative presentation of all studies. The work was supported by a grant from the LARS programme in Baden-Württemberg, Germany, (2005-08) which is gratefully acknowledged.

tasks and problems are easier understood by non-engineers. Roth (2003) visited the workplace of scientists in an experimental biology laboratory. The studies by Cardella and Atman (2005a,b) and Cardella (2006, 2010) are different since they did not observe workplace activity but the behaviour of mainly industrial engineering students when working on their final major project ("capstone project") as well as a few mechanical engineering graduate students. Bissell and Dillon (2000) reason about the use of mathematical concepts and models by control engineers based on their own experience.

Except for the study by Bissell and Dillon, all researchers used the so-called "ethnographic" approach, a qualitative research method (cf. Zevenberg 2000). They observed students and engineers working on a certain problem ("participating observation") and either recorded the activities or made notes. Moreover, they conducted interviews to learn more about the thinking processes and the attitudes and they investigated the documents and the tools that were used. From this material they developed interpretative categories in order to capture the way mathematical thinking or activities occur during the work on practical problems. In the rest of this section we provide an overview of the main results.

Embedding of mathematics: The mathematical concepts, models, and procedures are strongly embedded in application contexts. As opposed to educational scenarios, the main goal of practical work is to solve a certain problem or create something fulfilling specific requirements. Bissell and Dillon (2000) emphasise that model components and solutions are interpreted in application categories. Hall (1999) observed solution procedures which were based on mathematical models but simply became "convention" after some time. Roth (2003) recognised that scientists interpreted the graphs they created from their data by using application properties, not graph properties formulated in mathematical terms. Kent and Noss (2002, 2003) stated that they observed a wide spectrum of mathematics usage from the qualitative end to a detailed quantitative end, and particularly the qualitative arguments (e.g. a flow of forces through a structure) were dominated by application terms.

Mathematisation in breakdown situations: A reflection of underlying mathematical concepts and corresponding assumptions can become very important when the usual routine procedure does not work any longer or creates inexplicable results. Noss et al. (2000) coined the notion of "breakdown situation" if this happens. Gainsburg (2006, 2007a) observed such a situation where a usually applied model delivered results that required unrealistic additional structural elements for guaranteeing sufficient stiffness. This led to a re-consideration of the assumptions underlying the model. The mechanical engineering graduate students observed by Cardella (2010) had a simulation and a mechanical prototype as well. Since the prototype behaviour differed from the simulation, there was a need for explanation. Steevens and Hall (1998) and Hall (1999) describe a situation where current design sketches violated certain codes of practice which led to a reconsideration of the key figures used in the code. Even if there is no need for mathematisation caused by breakdown situations, it might be possible to make work more efficient by considering underlying mathematical concepts or models explicitly. Gainsburg (2005) states that a purely descriptive approach to workplace mathematics ignores this potential.

Role of modelling and models: Gainsburg (2005) states three kinds of activities related to mathematical modelling and models: Creation of new models for the situation at hand; selection and – if necessary – adaptation of existing models for solving problems; usage of situation-specific procedures and routines that avoid the explicit work with models. In her observation of structural engineers she recognised all of them. In the breakdown situation stated above the structural engineers first worked with two existing models which were both unsatisfactory and then tried to set up a combined model. The latter activity turned out to be too ambitious (it seemed to be rather a research and development activity that would have blown the schedule). So, the engineers stuck to their first model knowing about its inadequacy simply because it was generally well accepted such that they could justify their design decisions. This shows that a simple mathematical consideration is not sufficient. Bissell and Dillon (2000) who state that in their area of work engineers usually worked with existing models also emphasise that it is a matter of getting a consensus on which model to use. They see a hierarchy of three tasks and corresponding qualifications when using a model: to perform the mathematical manipulation work; to interpret the model and solutions computed within the model in application terms; to use the interpretation for giving recommendations and predicting behaviour. They consider the second one as the major part of work. Cardella (2010) also observed that the industrial engineering students set up a lot of models (cost model, model of a sorting process, model of a schedule table for the distribution of goods). She emphasises the role of uncertainty, estimation and iteration when the students tried to work within a model. The values of many model variables were unknown, so often estimations were necessary which were based on certain assumptions. They also had to choose some initial values and then to run through iteration cycles in order to get a satisfactory design (instead of using some sort of optimisation procedure). This kind of procedure is also confirmed by the study of Gainsburg (2007b) who recognised uncertainty and the need for estimation as a major challenge.

Role of tools and other resources: The predominant role of technological tools in everyday work has been recognised in several workplace studies (cf. Sträßer 2000, 2007). Bakker et al. (2004) and Kent et al. (2004) introduced the notion of "techno-mathematical literacy" in order to describe the ability to use technology and – as far as necessary – the underlying mathematical concepts and models for solving daily problems. Beside technological tools, Kent and Noss (2002) also consider other experts as additional resource. Communication with tools and with experts happens via "boundary objects". These objects reflect the understanding that is still necessary although much of the real mathematical work is done within tools or by specialised experts. From this perspective, the engineer is seen as a user at an interface who has to know what to expect, how to provide necessary input and to interpret the output. This understanding can be a priori or it can develop when using the interface frequently ("understanding through use"). Bissell and Dillon (2000) observed that in control engineering tools and techniques veered away from the strict mathematical notation to more graphical representations. They consider the adherence to the former in academic education rather as an act of retaining confidence.

Mathematical thinking: Cardella and Atman (2005a, b) used the five aspects of mathematical thinking identified by Schoenfeld (1992) in order to analyse and describe the mathematical activities of the students they observed. These aspects comprise:

- Knowledge base: content knowledge, knowledge of tools;
- Problem solving strategies;
- Effective use of resources: using and monitoring the use of resources like tools, documents, experts;
- Beliefs and affects: role of mathematics, own competence in using mathematics;
- Mathematical practices: mathematical approaches and procedures.

Cardella (2006) provides an overview of the findings concerning these aspects. In Cardella and Atman (2005b) the authors elaborate in more detail the problem solving strategies they observed. They emphasise two strategies: "guess and verify" and "separate the problem into smaller problems" which are quite general scientific strategies and, though it might be the case, it is by no means evident that proficiency in using these strategies was a result of the students' mathematical education. The resources the students used consisted of tools and experts, and they planned and monitored the usage. Cardella and Atman (2005a) found different beliefs regarding mathematics like "mathematics is only about content knowledge", "mathematics is a tool" or – more advanced – "mathematics is a form of thinking". Concerning the mathematical practices, they found – among others – the following ones: "having a mathematical perspective and a mathematical vocabulary", "dealing with uncertainty", "estimating". The latter two aspects were already described above and were also emphasised by Gainsburg (2007b).

Specific qualifications/challenges: Gainsburg (2006, 2007a) identified two particular challenges in the design work of structural engineers she observed:

- "understanding the phenomenon": Because of the design character of the work it was not sufficient to analyse an existing structure but to create a future one. So, it had to be checked which models were appropriate and which modifications were necessary, and for this an understanding of the requirements and characteristics of the new situation had to be developed bit by bit.
- "keeping track": the engineers used different models based on varying assumptions. Therefore, it was important to retain the overview and not to get lost in this multi-dimensional space.

Process of building up expertise: Gainsburg (2007a) investigated in which way engineering expertise was developed during the work of structural engineers. In her judgment, the essential task consisted of finding or developing an adequate model for a complex system in order to make and justify design decisions. The engineers tried and analysed several models for the given situation and in this process got a better understanding of their properties and restrictions. Although their thinking was clearly situated (they had to solve a concrete problem), it is the understanding of the models which is reusable in other circumstances and which makes up the expertise. This kind of knowledge is also important for understanding the computational basis of programmes that were used because it enabled the engineers to better judge their results.

Kent and Noss (2002a, b) observed that junior structural engineers often start by performing detailed quantitative analysis work whereas senior engineers rather do the design and use coarser qualitative models where mathematics is rather implicit. They state the hypothesis that their former computational work enabled them to develop a “feeling” for the behaviour of a structure. So, although they do not perform mathematical work any longer, it was an important step for building up the qualitative knowledge. This is also confirmed by Gainsburg (2007b).

Perception of the role of mathematics: Gainsburg (2007b) identified “engineering judgment” as essential qualification for the work of structural engineers. Among others, this judgment includes an appreciation of the role mathematics can play in the process. The overall goal consists of achieving an acceptable design, and acceptance is influenced by several factors like cost, official guidelines, or assumptions on loads. Mathematics is important and necessary for justification and verification but it is not sufficient. Many mathematical models and methods are available and there is still engineering judgment required to decide on which one to choose and which level of approximation to accept. Correspondingly, Gainsburg coined the notion of “sceptical reverence “in order to describe an adequate mathematical disposition of engineers towards mathematics. She considers this disposition as desirable for school students but this could also be transferred to engineering students.

3. Method of Investigation

In our studies we tried to capture the mathematical expertise of mechanical engineers. As was already observed by Kent and Noss (2002) for structural engineers, the group of mechanical engineers is by no means homogeneous. We restrict ourselves to those jobs in design or test departments (not research and development) where the majority of our practice-oriented students work after graduation. Nevertheless, one has to take into account that within these jobs one also has to cooperate with colleagues in other departments.

The ethnographic research method used by Kent and Noss (2002) and by Gainsburg (2006, 2007, 2008) was not applied because of the following problems:

- For getting a good understanding of the thinking processes intensive questioning of employees is required which demands too much working time from the employee.
- For a non-engineer it is very hard to capture the thinking processes and potential alternatives within a very restricted timeframe.
- The workplace and the current task of an employee might be not very representative.

We used a different method of investigation more similar to that of Cardella and Atman (2005a, b) mainly to overcome the first and second problem. We aimed at getting a very thorough and deep understanding of thinking processes such that we could also check for potential alternatives. For this, we wanted to be able to question people without disturbing them. Therefore, together with a colleague who worked in the car industry for several years and now teaches machine elements, design, and FEM, we identified four “typical tasks” our graduates might work on. They belong to different areas and are explained in more detail in the next section. For each task, we hired two students in their last (8^{th}) semester who worked on the task independently from each other (except for the last one) for 100 hours with payment. If possible, we chose students with different characteristics, one with more practical strengths and one with good results in more theoretical subjects. The students had industrial strength tools at their hands which are available at the university, i.e. CAD programmes (including FEM and mechanism analysis functionality), a machine element computation programme and a measurement configuration and processing tool. In this respect, there was no difference to an industrial workplace. They received a short task description and the colleague acted as mentor, so he played the role a team leader or senior engineer has in industry.

The students made notes on their working and thinking processes and problems as well as on the results. They also produced files using the industrial strength software. The author studied the documents and worked with the programmes himself in order to get a better understanding of the user interface and studied textbooks which are also used in lectures for acquiring additional background information. Based on this study, interviews with the students were conducted to clarify questions, explore alternatives and get demonstrations of the tool usage. Audio- and screen-recording enabled later investigation. Finally, the colleague involved was interviewed to clarify to which extent the approach and procedure of the students reflected the real work of junior engineers in industry. Moreover, we discussed which further improvements to the solution of the students would have been made in practice and where colleagues in other departments (in particular computational departments) would have been contacted.

The above material was used to answer the following questions regarding the mathematical expertise of mechanical engineers. The list is partially based on the results of the literature review stated above.[2]

- Which explicit (overt) mathematical concepts, procedures and practices were used? Which concepts were hidden?
- Which role did modelling and models or simple guidelines and rules play? Where on the spectrum from qualitative to quantitative reasoning could the work be located?
- Where were mathematical concepts embedded in applications?
- What kind of mathematical qualifications were necessary for using the tools (providing reasonable input, interpreting and using the output, making variations to improve the output)?
- Were there “breakdown situations” where a mathematisation helped or would have helped to resolve problems and to get further?
- What would the interface to other resources like experts look like (“boundary objects”)?
- Which aspects or competencies other than mathematical ones were important for successful work on the task and how were they related to the mathematical aspects?

The risks accompanying our method of investigation should also be mentioned:

- Unreal framework: There are no real customers and time pressure. The students were not part of a team and the feedback they received originated from a colleague. Many boundary conditions of the task which are normally discussed with customers or colleagues were not available and had to be thought up by the students or given to them by the colleague involved.
- Unreal tasks: It is not easy to find a task which can be handled without embedding in a real framework. Moreover, the task might not be representative for the usual work of a practice-oriented mechanical engineer.
- Non-representative students: All one can expect is that the students’ work is similar to that of junior engineers and even this might not be the case.

In order to minimise the risks the colleague involved in the project played an essential role. During the interview and later clarifying questioning he was asked about his judgment concerning the gap between a real working environment and the project framework. His judgment also formed the findings stated in section 5.

4. Tasks

The tasks that were chosen in strong cooperation with the colleague involved covered the following areas: static design, mechanism design, machine element dimensioning, taking and processing measurements. A graduate of our university might be required to perform one of these tasks in his or her job. Although – in this sense – the tasks are typical, this study does by no means claim to cover all such tasks by four representatives. In what follows we give a short description of the tasks and give an illustration of the results if possible. More detailed information can be found in Alpers (2006-2010).

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[2] The papers of Gainsburg and Cardella/Atman were available only after the first two tasks had been dealt with.

<u>Task 1: Static design</u>: Design a support for an ABS (automated breaking system) in a car where the installation space and connection points are given. Stress should be within the linear-elastic range; the first eigen frequency should be above 250 Hz. Mass should be low.

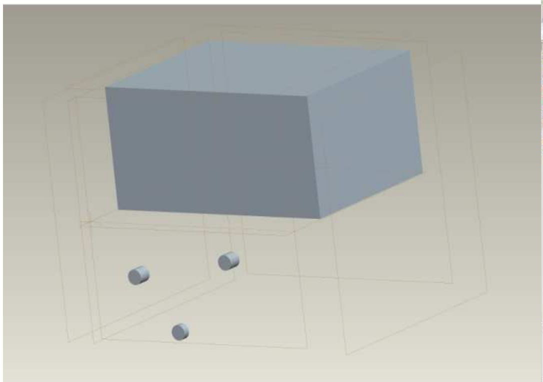

Figure 1: Design boundary conditions

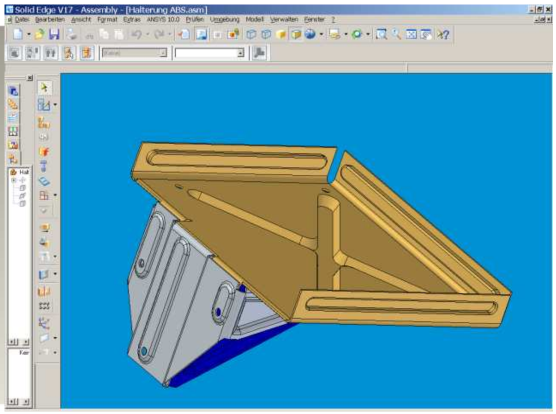

Figure 2: Solution of one student

<u>Task 2: Mechanism Design</u>: A knife has to be moved up and down for cutting off a part of a foil. This example is taken from a real machine for producing halogen lamps. Figures 3 and 4 show the sketch containing the boundary conditions as well as a solution of one of the students.

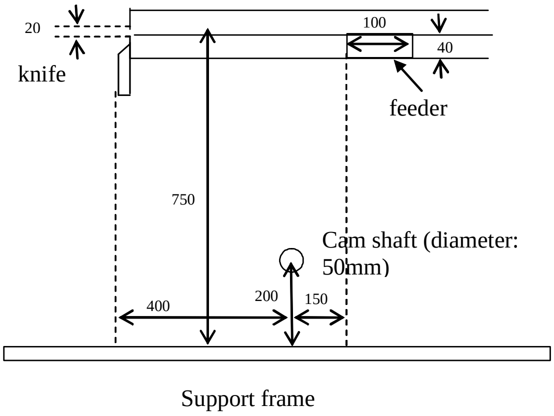

Figure 3: Boundary conditions

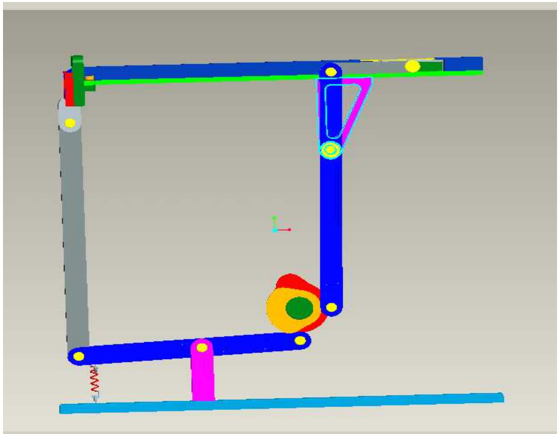

Figure 4: Solution of one student

<u>Task 3: Dimensioning of a gearing mechanism</u>: Dimension a gearing mechanism sketched in figure 5 where an input rotational velocity of 3000 rpm has to be transformed to approximately 1200 rpm. The input power amounts to 15 KW, the distance between shafts is appr. 200mm and life expectancy should exceed 10000hours.

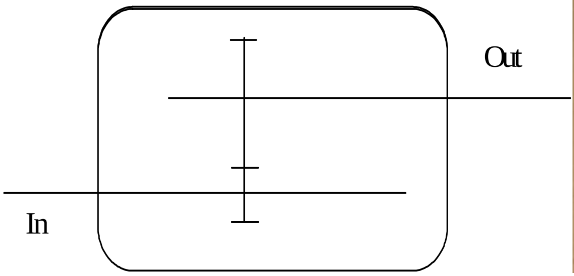

Figure 5: Sketch of gearing mechanism

Figure 6: Photo of an example

Task 4: Measurement task: Identify the critical components in a steering test bench (depicted in figure 7), measure the occurring strain using available measurement technology and data processing programmes and interpret the results.

Figure 7: Test bench for steering system

Figure 8: Steering track rod with strain gauges

5. Findings

We present our findings along the research questions formulated in section 3 which are based on the literature review but also leave space for additional insights. Detailed discussions of the single tasks can be found in Alpers (2006-2010). Here, we summarise and re-arrange the findings such that a more structured overview is provided. In the sequel we refer to the tasks by putting the task number in brackets.

Mathematical concepts – overt and hidden

Table 1 gives an overview of the mathematical concepts the students had to deal with when working on the tasks. Since we do not claim to be complete regarding the tasks, one cannot deduce that a mathematical concept is not important if it does not occur on the list. Nonetheless, it was quite evident that geometrical concepts, function concepts and algebraic equations were quite dominating. When we discuss the "boundary objects" at the interface of programmes the students used, we will describe in more detail in which form the concepts appeared.

Geometric objects and operations (CAD)
Geometric relations
Geometric configurations for gears (circles, angles, involute)
Functions with different representations (numeric, symbolic, graphic, by type with parameters), piecewise functions
Design of functions with certain properties
Derivatives
Continuity of functions and derivatives
Vectors in different representations
Vector decomposition
Prime numbers
Iterative algorithms
Equations and systems of equations, proportions
Algebraic formulae
Interpolation (linear, splines)
Curves, curve properties, offset curves
Diagrams

Table 1: Overt mathematical concepts

There were also concepts which were important in the background without being obvious for the students. Algebraic representations of geometric objects did not show up. Likewise, algorithms lying behind the geometric operations were invisible, e.g. when two surfaces were intersected or an offset curve was computed. In the FEM part just the load vectors appeared as mathematical concept whereas everything else (network of nodes, basis functions, system of equations) was hidden. In the mechanism part of the CAD tool all the routines for computing the positions, velocities, accelerations and driving torques are invisible to the user. And in the machine dimensioning part, the formulae used for computation were not depicted. Nevertheless, they can be found in the German norm (DIN) that was implemented.

<u>Quantitative Models and Modelling:</u>

There were different ways in which models and modelling activities appeared during the work on the tasks:

a) *<u>Geometric design</u>*: In the design tasks (1) and (2) the students had to create new parts instead of investigating existing ones as is usually the case in modelling tasks. In the static design task, the objects and operations for creation are offered by the CAD programme but it remains the job of the user to transform an imagination of the final shape into a sequence of geometric operations. During this process, a geometric object can also be determined by formulating relations (like orthogonality or parallelism). The latter makes a geometric configuration quite flexible since certain properties can be changed (e.g. the length of a side) without distorting other properties and hence the principal shape of the object is preserved. In the mechanism design task, beside the geometric design there is also a mechanic design part since the kinematic structure including swivel joints and prismatic joints must be specified.

b) *<u>Modelling situations with available (existing) modelling means</u>*: In the machine element dimensioning task (3), the students had to set up a model for the forces acting on the shaft and to split them up in order to use the shaft computation part of the machine element dimensioning programme. Here, the modelling means (forces) are quite clear and the challenge consists of using them appropriately. For this, an understanding of mechanics is important which seems to be similar to the notion of "understanding the phenomenon" stated by Gainsburg (2006). A similar situation can be found in the measurement task (4) where the students had to set up a model (equations) for the overall strain that occurred in the full bridge, and it was clear that for doing this one had to investigate the single strains resulting from particular strain gauges. So, the interesting deviation from what can be usually found in the literature on modelling is that the modelling means (interesting quantities, ways of combining these) are already known from mechanics education.

c) *<u>Finding an appropriate existing model</u>*: In the machine element dimensioning task (3) there were different models and corresponding algorithms for computing the occurring stress depending on the load situation, so the first activity consists of identifying the characteristics of the situation in order to choose an appropriate model. This can be considered as another facet of the challenge of "understanding the phenomenon" observed by Gainsburg (2006). A similar setting could be found in the measurement task (4) where the identification of the load case is important for choosing the right bridge arrangements.

In the mechanism design task (2), one had to set up a motion function for the knife as piecewise defined function (there is an upper rest and a lower rest, and the motion between these resting phases has to be designed). For this task, candidate functions are offered in a respective guideline published by the German Association of Engineers (VDI), so one can choose one of the function pieces presented there to connect the resting phases. The guideline also contains criteria which help to decide which function to choose.

d) *<u>Work within existing models</u>*: In the machine element dimensioning task, there is an algebraic model (consisting of several cascading formulae) for computing the occurring stress (on a tooth) on the one hand and the allowed maximum stress on the other hand and the quotient of the latter by the former gives the security factor (for a more detailed description cf. Alpers 2008). The model contains many quantities for which first of all initial values have to be found. For doing this, there are sometimes rough calculations. For other quantities, estimates for initial values come from experience, i.e. an engineer remembers dimensions of similar devices. Usually, the first "design" is either over- or undersized such that several iterations are necessary to achieve a satisfactory design. In order to make goal-oriented variations one has to have some goal or goals in mind like minimum weight (or as an overarching goal: minimal costs). For performing such variations, one has to judge the influence of quantities occurring in the model. This judgment can be of algebraic nature or it can be rather qualitative. The user of the model must be aware of the uncertainty that is inherent in some factors modelling the application situation. For example, there is a so-called “application factor” which roughly models additional stress caused by problematic environmental conditions. The user should be aware of how this uncertainty influences the resulting security factors.

In the cascading model there are several intermediate stages of stress computation and it is a real challenge to keep track of the kind of stress one is currently dealing with. Such an observation was also made by Gainsburg (2006).

The students did not know exactly how parts of the model had been originally derived from more basic principles but they had to use it thoroughly. According to the colleague involved this happens often in industry where there is not enough time to go back to the basics. Therefore, working diligently through a model that is just partially understood is also an important qualification.

Application of rules and qualitative reasoning:

Rules and qualitative reasoning play an important role for finding an initial configuration in design work. If there is no fixed model available, construction rules or experience (knowledge of parts with similar functionality) are used to get from a merely imagined “picture” to an initial part design. Such a rule in CAD might be to start with a comprehensive block and then to remove material successively. Sometimes (in particular in machine element dimensioning tasks), there are rough calculation models to estimate an initial value based on the given load situation. Sometimes, it is again experience with similar situations that helps. For a junior engineer who has not seen many machine parts in different situations it is always challenging to find reasonable values to start with and he or she will probably need more iteration cycles afterwards to get to an acceptable final design. This shows the importance as well as the restrictions of experience since it is mostly bound to comparable cases. Rules might also be available for choosing an appropriate model as is the case with the VDI guidelines mentioned in the previous item which suggest function types for motion design and provide criteria for usage.

When the initial design is changed, qualitative thinking plays again an important role. For example, if a FEM analysis shows that stress in certain regions of a part is too high, then material should be added (without computing “how much” or “where exactly”). For increasing the lowest eigenfrequency, reduction of mass remote from an axis of oscillation is an appropriate means, and in order to enlarge stiffness fins have been used in task (1). A more general meta-rule recommends to change just one quantity when making variations in order to have a clear understanding of cause and effect. Qualitative thinking was also applied when variations were made in the machine element dimensioning task. For example, the breadth of a tooth has a positive influence on the maximum bearable stress. This relationship can be deduced from the quantitative model that is used for computation but with the students it seemed to be rather a piece of qualitative knowledge.

When the students looked for the most vulnerable components in the steering mechanism (4) they did not set up a model of the mechanism but rather used the very coarse rule “When the cross section is small, it might fail”. According to the colleague involved, in real engineering life the model would have been finer and quantitative or a hypothesis concerning the critical components might have come from a simulation run.

It is an interesting question whether knowledge of quantitative models makes variation more efficient. Efficiency is an important criterion since often engineers have very tight deadlines. In our studies we observed much qualitative thinking when students made variations, and they had to make just a few in order to obtain an acceptable result (under the circumstances of the project). In a few instances we observed that having a quantitative model would have made variation more efficient, e.g. having a power model in the mechanism design task (2) in order to recognise the most important factors influencing the necessary maximum torque of the driving motor. The interview with the colleague involved revealed that in practice there are two quite different situations. In the so-called “special purpose machinery manufacture” where customized machines are designed and built such that nearly all machines are different, quick solutions are predominant and hence qualitative reasoning with just a few iterations is preferred. This is what we observed in the students’ work. In serial production, in contrast, where the same parts are produced very often (as is the case in car industry, e.g.), it is quite worthwhile to spend more effort for setting up more precise quantitative models and to perform mathematical optimisation within these models in order to realise the “final 5%” of optimisation potential. So, one should be very careful when generalising from the procedures observed during the project.

Finally, qualitative knowledge was also important when results were interpreted and checked for validity. In the design task (1), the students observed after the FEM run that the stress near one of the clampings was quite low (visible by deep blue color). But they knew as a rule that at such places the stress should be rather higher than average. This enabled them to recognise that the clamping had not been identified as such. Since specifying a clamping was done by pointing at a small part within a larger construction (see figure 1), this is quite prone to error, so without a way of checking it is very likely for mistakes to happen and to remain unrecognised.

Qualitative thinking was also observable when interpreting results like functions and curves. Functions were considered in terms of "going up" or "going down" in certain regions, or being smooth or oscillating. Such qualitative statements are important when results have to be compared with application behaviour as will be discussed below.

<u>Embedding in application contexts:</u>

Mathematical concepts, objects and procedures appeared as embedded in application contexts in several ways. First, some mathematical objects were completely embedded in application objects, e.g., pre-defined objects in the CAD system like chamfer and fin which are parameterised geometric constructs for which just the parameter values have to be input. They are offered by the programme in the so-called sheet metal mode. For using them properly one has to understand the geometric meaning of the parameters.

Another example occurs in the mechanism design part of the CAD system where for the driving component motion functions had to be provided: either position or velocity or acceleration. Then, the other functions could also be shown on screen graphically. So, there was no general concept of function and derivatives visible but just the application meanings. Nevertheless, knowing these concepts helped in finding input problems as will be described below.

Secondly, application meaning turned out to be important for performing operations on mathematical objects. With fins in the sheet metal mode, the bending operation was no longer available. Although mathematically this could be performed, it does not make sense in real bending machines since the material has to "flow" somewhere.

Thirdly, application meaning was important for the interpretation of mathematical results. When interpreting the strain curve in the measurement task (4) one had to rotate the steering wheel "virtually" and to interpret the strain correspondingly. A similar situation could be found in the mechanism design task (2) where the necessary driving torque of the motor had to be considered over one rotation.

The above observations confirm the statement by Bissell and Dillon (2000) who claim that engineers using mathematical objects predominantly think in application terms.

<u>Tools: Input/output objects (boundary objects), underlying models, and practices of use</u>

The usage of software tools was an essential part of work in nearly all tasks. Only in the measurement task (4) was tool usage of rather minor importance. In the following, we will first present the mathematical concepts (boundary objects) that were directly visible at the user interface (as input or output objects). Then, we will discuss in how far internal mathematical models and procedures were important for efficient and effective use, and finally we describe the practices of use we observed.

When using a CAD programme, the students had to deal directly with geometric objects and operations. But there were no algebraic representations as are usually taught at school and university (like ax+by+cz+d=0 for a plane) but only visual representations on screen which have some properties (attributes) that might be set. The same holds for the available geometric operations. The students had to anticipate a sequence of operations to create a desired object from basic ones. Also visible were geometric relationships which can be used to determine a sketch. For many students it is non-trivial to find relationships in a way that the resulting geometric figure (like a hexagon) can be scaled (retaining the relationships).

The FEM programme attached to the CAD programme (i.e. the version for designers) needs as input the specification of load vectors and provides as output stress distribution (comparison stress). Moreover, eigenfrequencies can be computed.

The mechanism part of the CAD programme requires as input the motion function of the "driving" part (e.g. a motor). In a backward computation, this can also be the desired motion function of a driven part which then acts as driving part. The functions can specify position, velocity or acceleration. Different representations of functions are offered as stated above. The output also consists of functions (numerically or graphically), e.g. motion functions of the driven part or functions concerning other mechanical quantities like torque over time. Beside functions, there are also motion curves which are e.g. required for constructing the boundary curve of a cam disk. These curves are constructed by using a certain number of computed points and interpolation. The user just sees the resulting curve which might be problematic as will be discussed under the heading of “breakdowns” below.

In the machine element dimensioning programme three parts were used: the module computing the actually occurring and the allowed stress in gears (tooth base and tooth flank); the module computing the maximum stress in shafts; and the module computing the maximum force and the life expectancy of bearings. The first module needs and provides a lot of geometrical data. The help text shows a picture of the underlying geometric model. The programme gives no information on the underlying computation but just offers different variants of computations (according to a DIN norm or according to a widely used text book). The output of the first module cannot be used immediately for feeding the second one, so one has to do some intermediary computations oneself. Therefore, simple delegation to a computational programme is not possible. Some of the values needed as input have to be retrieved from diagrams where several curves are depicted (for different values of a certain quantity, a way to make use of a function of two variables in a two-dimensional picture). Here, it is necessary to read the diagrams correctly in order to retrieve the right value.

In the measurement configuration programme the model of a full bridge is required to connect and configure the measuring equipment correctly. The programme provides the desired strain values directly without further computation (so the voltage measured is transformed into strain). The strain values depend on the steering angle and from the output of tabulated values the user has to build a curve for visually interpreting the results which was done in Excel®.

In the sequel, we will discuss the practices of tool usage supported by the tools themselves as well as the actual practices that could be observed with the students. We will particularly address the question in how far mathematical expertise is still necessary for effective and efficient work with the available tools.

- Avoidance of formal mathematics: In some tools or tool parts it is quite obvious that the tool designers aimed at avoiding formal mathematics as far as possible. This was the case in CAD where geometric objects and operations dominate and nearly no algebraic representation shows up. Even more noticeable is the hiding of any mathematical models and procedures in the FEM module for designers that was attached to the CAD programme. In those programme parts of the CAD system that dealt with mechanisms, though, this was simply not possible since motion cannot be specified by drawing the mouse pointer. In the machine element dimensioning programme, the computational part is also largely hidden.
- Correct specification of tool input: In nearly all programmes it was sometimes very easy for the user to provide wrong or inadequate input leading to equally wrong output. In the machine element dimensioning tool output of one part could not be directly used as input for the next but it was easy to simply do this and hence get at least slightly different results (names of forces were very similar but the forces were different). When feeding the mechanism tool with discrete data on the desired knife motion, the rounded data (4 significant digits) was too coarse such that numeric differentiation led to erratic velocity and acceleration functions (see the item on breakdown situations below). In the FEM part students thought that they had specified a part of the geometry as bearing but that was not the case, and the programme did not make this obvious to the user.
- Initial design and variation: In the discussion on quantitative models we already stated that tools give very limited support for finding initial values but are very important for efficient variation later on in an iterative way.
- “Understanding through use” was one of the interpretations of tool usage given by Kent and Noss (2002). We also observed behaviour which can be interpreted this way. In the CAD programme, for example, the operations for rotating, moving and zooming were not executed by thinking about rotation angles or matrices but just by trying the respective mouse facilities and getting accustomed to the operations. It might be questionable what “understanding” under these circumstances means. This is certainly restricted to knowing the effects of operations without knowing how the operations can be formalized and realized. For many

situations this will be a sufficient form of understanding. Only when problems come up and the routine handling does not work any longer or produces strange results, a deeper understanding of what lies behind the operations might be necessary to proceed (see the item on breakdown situations below for an example). If this understanding is not available, because the underlying mathematics is too complicated for the average design engineer (not having a special computational education), the user has to find a work around or ask an expert. When one of the students used the FEM programme in the design task (1) he had problems to specify the force application point within the geometry. He could not understand why the programme did not accept the geometric part he had pointed to. He got around the problem by adding another small piece in the geometry which was irrelevant for later production but allowed him to have the place specified approximately. According to the colleague involved this shows an important qualification in tool use, i.e. to flexibly find acceptable work arounds when encountering tool intricacies.

- Finally, given the risks described above, it is very important to be able to check intermediary and final results for validity if possible. This can be done by using rough qualitative knowledge as in the case of checking the stress output of the FEM programme where students observed that there was low stress at a certain bearing contradicting the rule that stress should be higher in such regions. In case of the mechanism task resulting torque could be checked by a rough quantitative power balance model. Such a quick check is not always possible. When the geometry of an object gets too complicated then there are no simple models for comparison. In such cases simulation results are checked by performing measurements like the one in the measurement task (4).

The above points show very clearly that a simple delegation of tasks and problems to software tools is by no means possible. Though the tools enable users to create and vary designs and configurations very quickly and thus open new ways to work, understanding the background models is still important for providing correct input, working with the tool in an efficient way and checking the results for validity. A further point is trust which is also mentioned in literature. In order to be content with the results e.g. of the machine element dimensioning programme the students went again through an at least coarse version of the computational algorithm given in a text book to have confidence in what they did and in the programme output.

Breakdown situations

The students encountered breakdown situations most prominently in the mechanism design task (2) (see Alpers (2010) for more detail). One of the students was confused by the cam disk boundary curve the mechanism tool depicted on screen. He expected that when the motion of the knife had a rest phase, the corresponding part of the boundary curve should be a part of a circle which was not shown on screen. So, it was first of all his geometric understanding which made him expect a certain output. But he could not clarify the issue. In order to see the reason and change the tool use correspondingly, one has to know that the tool computed 25 positions and used spline interpolation. The interpolated curve had more curvature in the part under consideration and less in other parts since spline curves usually interpolate rather smoothly. So, here much more positions were required for adequate interpolation which took more processor time and hence was not the default setup.

Another breakdown situation occurred when the torque function computed by the tool oscillated heavily which could not be explained by the real situation. So, we had a look at the input to see whether the problem showed up already there. Although the graphical representation of the motion function table that was given as input looked reasonable, a further look at velocity and acceleration (also offered in graphical form by the tool per mouse click) revealed that here already the strange oscillatory behaviour could be observed. This was due to the coarseness of data (precise to 4 digits) and to numeric differentiation. When we then provided more precise data (7 digits) the output looked as expected. Here again, there are two qualifications: first to generate an expectation at all, and second to be able to find out why it is not met.

Interface to experts:

There is not just an interface to tools but also an interface to human experts where a common understanding of mathematical concepts and procedures “at the boundary” is important. The insights formulated below originate from discussions with the colleague involved who provided information on when and what about experts might be consulted by a practice-oriented engineer working on the tasks. With experts we mean computational engineers working in computational departments of larger companies or in specialised engineering consulting firms.

In the design task (1) it could be important to reduce weight because this criterion plays a major role in car design. For this purpose, there are special topology optimisation programmes that are normally handled by computational engineers. These programmes remove material in uncritical regions and add it in critical ones in order to minimise weight. A practice-oriented engineer should know about the existence of such tools but usually will not have any background on the available optimisation algorithms and their benefits and pitfalls. The communication between the practitioner and the computational engineer will be concerned with the restrictions and the optimisation goal ("objective function") on the one hand and with acceptability of results on the other hand. In the first case the computational engineers will rather get the necessary information by questioning the practitioner correspondingly. When it comes to judging the results of an optimisation run, the practitioner has to keep in mind the boundary conditions of real production (costs, availability) and adapt the output. The discussion between the engineers will deal with geometric shapes and bodies, in particular the simplification of free-form geometries produced by optimisation runs and the effects thereof.

Concerning the FEM computations the practitioner might contact a computational engineer when results are inexplicable. He or she should at least know that problems can turn up when using approximative methods like FEM. The computational engineer's view is much more detailed including the network of nodes and polynomial basis functions but discussions will rather deal with regions showing problematic results and the computational engineer then will take the necessary measures to set up the tool (or better: a version of the tool directed at computational engineers) adequately. The practitioner should also know that the version available in his/her familiar CAD environment can only deal with linear-elastic situations and that an expert is to be consulted if larger permanent deformations occur.

In the mechanism design task (2), modelling of mechanisms in specialised multibody simulation tools might be necessary when results provided by the simple mechanism part in the CAD programme seem to be problematic. The mathematical objects which are interesting at the communication interface between engineers are similar to those appearing at the mechanism tool interface: functions and curves.

For machine element dimensioning (3) there are also specialised programmes that implement more detailed models and algorithms. Here again, the practitioner should be aware of the restrictions and uncertainty of the simpler programmes normally used. The expert who knows the more sophisticated models will question the practitioner in order to acquire the input data needed by these models which will in general describe the application situation in more detail.

In the measurement task (4) the communication quite often works the other way around: Computational engineers use simulation packages in order to compute stress for complicated geometries and/or load cases where there is no possibility to check results roughly by using coarse models. Therefore, they want to get (partial) confirmation by having measurements taken. At the communication interface the engineers use not only the one-dimensional stress/strain model necessary for task (4) but usually talk about two or three-dimensional stress/strain models using stiffness matrices and vectors. Here, a detailed knowledge is required on both sides of the communication interface.

<u>Keeping track of other aspects:</u>

When working on the tasks the students' thoughts were not restricted to mathematical or mechanical models but many other aspects were also taken into account as is the case in normal engineering life. The overwhelming goal of all technical design work consists of finding a cost-effective acceptable solution. Keeping costs in mind includes a variety of aspects where experience and broad background knowledge play an important role:

- Are the parts occurring in the design available on the market, are they cheap "normed" parts or are they custom-made which makes them more expensive? Over-dimensioning might be even cheaper in the end when using normed parts.
- Which material should be chosen? Material with better stiffness properties might lead to larger costs than using more material of lower quality.

- How many different parts occur in the design? Again, over-dimensioning (choosing the larger size for all screws, for example) can be cheaper in the long run because of additional logistic costs caused by having many different parts on shelf.
- How expensive is the production of the parts occurring in the design? Which kinds of machines are necessary for production. Sheet metal bending could be considerably cheaper than producing parts with free-form geometry in milling machines.
- How easy is it to assemble the parts in order to set up the whole machine? Can assembly be automated or is still hand work necessary?
- What is the additional cost of developing a more sophisticated solution using more time, software and hardware and maybe human expert resources? Is the customer willing to pay the additional costs?

The above list and the other findings stated in this section show that there is a variety of aspects to be considered for finding an acceptable solution among which functionality and costs are most important. It is particularly challenging for an engineer to keep track of many aspects. Mathematical models help with some aspects, in particular functionality, but there is much more to take into account.

6. Conclusions

The method of investigation allowed us to probe deeply into the ways mathematical concepts were used when the students worked on the tasks. The nearly permanent availability of the students and the colleague for questioning helped to clarify many situations and tool usages. It also made it possible to think about alternative ways. The colleague involved played a decisive role for checking whether the work resembled that of junior engineers in industry. In our findings we took his judgments into account. Our approach has certainly its limitations in that it does not capture the rich work environment (embedding in larger project, project teams, involvement of different departments, customer requirements, etc.) at many workplaces. It also covers just some typical tasks for a practice-oriented engineer. Therefore, the results provide information on some engineering work, but by no means on all engineering work. Nevertheless, the method can be used for further studies and the findings can also serve as a set of hypotheses when visiting real workplaces. There, one could investigate in more detail the communication interface to experts, e.g. by analysing requirement specifications for services to be delivered by computational departments.

Although computational tools permeate engineering work more and more, a simple delegation of mathematical work to tools is not possible. Mathematical concepts are still important at the interface of tools and an understanding of the concepts is necessary for providing correct input. Engineers work with both quantitative and qualitative models for finding initial designs and for varying such designs. The studies showed that although the students often applied qualitative thinking, the use of quantitative models might have made work more efficient in some instances. Having or generating a mathematical expectation of results to be provided by a tool also helps considerably in checking the output, detecting strange situations (breakdown situations) and overcoming them.

In the tasks, students mainly used already existing models which first had to be found. The work within these models then aimed at finding an acceptable design. This result is in line with the statements by Bissell and Dillon (2000). Even if the students did not understand every aspect in detail (some parts are based on research work), they had to go through the model diligently. This shows that even the often “despised” computational work with just a partial understanding has its value. If the students had to set up new models then the modelling means (important quantities and relations) were already known from mechanics or machine element literature.

Finally, engineering work is not restricted to work within mathematical models since there are many other aspects to be taken into account. Mathematical education should correspondingly aim at generating an attitude of critical appreciation (or in Gainsburg’s words: “sceptical reverence”) of the role of mathematics in engineering work: It helps in working with and without tools effectively and efficiently but it constitutes just one major aspect of engineering life.

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