Modelling as a big idea in mathematics – Knowledge and views of pre-service and in-service teachers

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Abstract
Modelling is agreed to be a big idea for mathematics as a scientific discipline with high relevance for mathematical literacy. Consequently, teachers should be aware of this big idea and know how modelling relates to a variety of curricular contents. However, especially quantitative empirical research into knowledge and views of pre-service and in-service teachers related to modelling is scarce. The study presented in this paper – based on a paper and pencil-test – concentrates on professional knowledge and views of Austrian pre-service teachers about modelling as a big idea and contains first exploratory comparisons with in-service teachers. The results suggest that especially for a sub-group of teachers there is a need of professional development related to modelling. Taking an outlook on these findings empirical research into the role of the views examined here for the teachers’ choice of learning opportunities in the classroom and into the structure of professional teacher knowledge related to modeling seems to be necessary.

Keywords: modelling, big idea, professional knowledge

1 Introduction
Modelling is considered as a big idea for mathematics and mathematics education by the corresponding scientific communities (cf. Blum et al., 2007; Lesh et al., 2007; Stillman et al. 2008). In contrast, empirical findings have suggested that mathematics teachers might lack awareness of the significance of modelling in everyday instruction. Even though such findings suggest deficits in the teachers’ knowledge related to modelling, relatively little is known about such professional knowledge from a quantitative empirical point of view. Hence, this paper responds to this research need by investigating views of Austrian pre-service teachers about the significance of modelling and related professional knowledge, including first exploratory comparisons with in-service teachers. The findings highlight differences among the teachers, suggesting that sub-groups of teachers in both sub-samples have deficits in professional knowledge components necessary for an awareness of modelling in the classroom.

The paper consists of an introduction to the theoretical background, research questions, information about the sample and methods, empirical results and their interpretation in a concluding section.

2 Theoretical Background
Modelling as a big idea in mathematics

Mathematical modelling affords reflecting on mathematical concepts and their roles for solving real-world problems as well as creating rich learning opportunities. As Pollack (1979, p. 240) states, modelling “requires an understanding of the situation outside mathematics and of the process of mathematisation as well as of the mathematics itself”. Conversely, the role of mathematics can be recognised particularly clearly when reflecting on modelling processes. The contribution of mathematical ideas and concepts (Lakoff et al., 2000) can be made transparent for learners through their use in modelling real-world problems. A prototypical example of the impact of modelling on
mathematics with implications also for mathematics instruction can be seen in De Lange’s (1996) approach of using real world problems as an opportunity to develop mathematical concepts. Further, the approach of developing mathematical knowledge by using and refining models not only allows curricular conceptions but also helps to anchor conceptual knowledge in everyday thinking (cf. Blum et al., 2007; Lesh et al., 2007; Stillman et al., 2008; Van Dooren et al., 2009). These considerations related to modelling suggest that modelling is a mathematics-related big idea. According to the approach of the project AB-Cmaths (‘Awareness of Big Ideas in Mathematics Classrooms’, www.abcmaths.net; Kuntze et al., accepted), we refer to the notion of big ideas in mathematics in the sense of ideas that should

- have a high potential of encouraging learning with understanding of conceptual knowledge (including linking and anchoring of knowledge)
- be relevant for building up knowledge about mathematics as a science
- support abilities of communicating meaningfully about mathematics
- encourage reflection processes of teachers connected with designing rich and cognitively activating learning opportunities as well as with accompanying and supporting learning processes of students.

Hence we do not consider big ideas as belonging to a pre-defined catalogue, but we emphasise the potential of these ideas for teachers when they design learning opportunities for making mathematics meaningful and for facilitating rich conceptual learning according to the aspects above. This pragmatic understanding affords integrating prior approaches to overarching concepts, like e.g. to “fundamental ideas” (Bruner, 1960; Schweiger, 2006). Examples of such big ideas beyond modelling are presented in Kuntze and colleagues (accepted, cf. also ABCmaths team, in preparation).

**Implications for the classroom and desiderata for the professional knowledge of mathematics teachers**

Today the importance of mathematical modelling is acknowledged in current curricula and standards. (e.g. NCTM, 2000; AECC, 2008). Consequently, modelling is given a key role in the assessment of mathematical competency. For instance, it is considered as a key component of mathematical literacy (OECD, 2003).

For fostering competencies related to modelling in the classroom, students should be encouraged and supported to engage in meaningful modelling tasks and modelling activities (e.g. Martinez & Brizuela, 2009). As a precondition it is acknowledged that students learn mathematical modelling by (actively) participating in the experience of constructing a model and working to correct and improve their models. This requires teachers to allow students to formulate, test, discuss and adjust their thinking. The role of teachers is to provide opportunities for this learning to take place and to guide the students through a modelling process, while allowing them freedom within each stage.

As a prerequisite for teachers to design corresponding learning opportunities with rich modelling activities, teachers need to be aware of the idea of modelling and its relations to a variety of contents. Hence, teachers’ views about the significance of modelling appear as crucial for the development of mathematics instruction. Such views should have a backing in corresponding professional knowledge (Shulman, 1986), that connects different mathematical content domains through the specific idea of modelling, e.g. through meta-knowledge about modelling as presented in the modelling cycle (e.g. Greefrath et al., in press).

**Professional knowledge and views of mathematics teachers about modeling**

Studies into teachers’ views about modelling have to be seen as embedded in more general research focusing on professional knowledge and teachers’ beliefs or convictions (e.g. Shulman, 1986; Ball, Thames & Phelps, 2008). The corresponding theoretical background is developed in more detail in Kuntze (in press). Leading to the research questions of this study about the significance of modelling as a big idea and professional knowledge related to this idea, we will, in the following, focus on specific findings. A recent study about teachers’ views about modelling has been realised in the framework of the LEMA project (Maaß & Gurlith, 2009). In this study, teachers demonstrated little knowledge about modelling, e.g. about the modelling cycle. Kuntze (in press) showed that pre-
service teachers favoured tasks with low modelling relevance to tasks with higher modelling requirements. German in-service teachers showed a tendency of similar patterns of answers in a task-specific survey centred on the idea of modelling. Moreover, the in-service teachers were not very confident with respect to their knowledge about the modelling cycle. Siller et al. (accepted) focus on the importance German and Austrian pre-service teachers associate with modelling in comparison to other important mathematics-related ideas. The results of a cluster analysis in the framework of this exploratory prior research indicate that a large part of the pre-service teachers saw modelling as relatively insignificant compared to other big ideas. Consequently, we expect that teachers might attribute not the highest significance to modelling. The present study aims to explore whether such views coincide with a non-optimal professional knowledge related to modelling, to broaden the prior study and to include a sub-sample of in-service teachers.

3 Research Questions

As empirical evidence related to teachers’ knowledge and perceptions about modelling as a big idea is scarce, this paper concentrates on (A) professional knowledge about modelling and (B) on pre-service teachers’ views about the significance of modelling. Exploratory comparisons with a relatively small sample of in-service teachers will be included. Hence, the research questions are as follows:

(a) Are pre-service and in-service teachers able to connect contents through the big idea of modelling and do they have meta-knowledge about the modelling process?
(b) Which significance do pre-service and in-service teachers assign to the big idea of modelling and how do they see this idea related to specific content areas?

4 Sample and Methods

In order to find out about the research questions (a) and (b), a paper-and-pencil-test and questionnaire was administered to 39 Austrian pre-service teachers (30 female, 8 male, 1 without data, mean age 23.5 years; SD=3.5 years) and 11 Austrian in-service teachers (5 female, 5 male, 1 without data, mean age 32.5 years; SD=9.6 years), working for on average 5.8 (SD=9.1) years at academic-track secondary schools.

The questionnaire contained two test items about connecting contents through the big idea of modelling (for a sample item see Fig. 1), one question about the modelling process, a questionnaire unit about the perceived significance of big ideas, a rating of tasks with modelling requirements (cf. Kuntze, in press), as well as questionnaire sections on views related to modelling and on global instruction-related convictions. According to the research questions, we will focus on the first parts in the following.

Figure 1. Sample item (connecting contents through the big idea ‘modelling’)

For gaining an overview on the quality of the teacher’s answers to the items such as in Figure 1, a top-down coding method was used. The coding categories concentrated on the aspects of the existence of a codable answer, on the quality of the answers, on the transfer level of the examples, and on the embedding of these examples.
5 Results

Research question (a) focuses on professional knowledge related to connecting contents through the big idea of modelling. In both items, teachers were asked to give examples related to aspects of modelling (s. Fig. 1, which corresponds to ‘task 2’ in the following analyses). The frequencies of adequate examples given by the teachers are displayed in Figure 2. Between one fifth and more than one third of the teachers did not provide any adequate example, even if a first example was already given in the items, respectively. The influence of these given examples on the examples provided by the teachers was not high, as can be seen in Figure 3: Figure 3 shows the frequencies of the transfer level coding, which distinguished between examples close to the given example vs. examples in other content areas. The results show that a big majority of the examples was related to other content areas.

Figure 2. Numbers of adequate examples given by teachers (big idea ‘modelling’)

Figure 3. Transfer level of examples given by the teachers

Table 1. Phases of the modelling cycle

<table>
<thead>
<tr>
<th>Percentage of teachers who mentioned phases of modelling process</th>
<th>pre-service teachers</th>
<th>in-service teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Situation</td>
<td>86.4%</td>
<td>100%</td>
</tr>
<tr>
<td>Model of the situation</td>
<td>27.3%</td>
<td>25%</td>
</tr>
<tr>
<td>Model of the problem</td>
<td>27.3%</td>
<td>100%</td>
</tr>
<tr>
<td>Mathematical model</td>
<td>86.4%</td>
<td>75%</td>
</tr>
<tr>
<td>Mathematical result</td>
<td>72.7%</td>
<td>100%</td>
</tr>
<tr>
<td>Interpreting the mathematical result</td>
<td>54.5%</td>
<td>100%</td>
</tr>
<tr>
<td>Validating the interpreted result</td>
<td>27.3%</td>
<td>100%</td>
</tr>
<tr>
<td>Running through the cycle another time</td>
<td>4.5%</td>
<td>25%</td>
</tr>
<tr>
<td>Use of technology</td>
<td>0.0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Figure 4. Ratings of the significance of big ideas
A global result concerning the question about describing phases of the modelling process was that 43.6% of the pre-service teachers did not provide any answer at all. Among the in-service teachers, 63.6% did not give any answer to that question. For the remaining 56.4% resp. 36.4% who gave an answer to the question, it was coded which phases and aspects of the modelling process were contained in the teachers’ answers. Table 1 shows the corresponding frequencies. These results indicate that some of those pre-service teachers who answered this question did not describe all phases of the modelling process. For the in-service teachers, about two thirds did not describe the modelling process at all, but those who did, showed a rather detailed knowledge about the modelling cycle.

Research question (b) concentrates on the significance assigned to the big idea ‘modelling’ by the pre-service and in-service teachers in comparison with other big ideas. Figure 4 presents the corresponding results of the teachers’ ratings (ratings by numerical values from 0 (low significance) to 5 (high significance)). As the data show, the big idea ‘modelling’ was rather not given the highest significance by both sub-samples, replicating the basic tendency of a prior study (Siller et al., accepted) with more than 100 German and more than 40 Austrian pre-service teachers.

This perception of the significance of modelling for the mathematics classroom might have to be seen against the background of the results displayed in Figure 5: These results concern the perceived significance of modelling for selected content domains. The diagram indicates that the significance of modelling might – in the eyes of the participating teachers – be restricted to a rather narrow field of curricular contents. However, for three obviously relevant content areas, the teachers do on average see a high significance of modelling.

![Figure 5. Perceived significance of modelling related to content areas](image-url)

### 6 Discussion and Conclusions

The results related to research question (a) suggest that there are differences within both sub-samples: Some pre-service and in-service teachers obviously have the professional knowledge necessary for connecting contents according to the big idea ‘modelling’, whereas other teachers did not even give one example of a content relevant for modelling. Likewise, such differences were also observed for meta-knowledge about the modelling cycle. Even if the results should be interpreted with care given the relatively small sample, the evidence suggests that among in-service teachers there might be a big gap between those who are skilled in modelling issues and others who lack of such knowledge concerning the big idea ‘modelling’ at all. As far as the pre-service teachers are concerned, the awareness of modelling and corresponding professional knowledge should also be developed further. For the ABCmaths project work and beyond, the results suggest supporting teachers’ professional development by specific learning opportunities connected to modelling.

As views about the significance of mathematical modelling can be seen as an additional indicator of the awareness teachers have towards modelling as a big idea when defining learning goals...
and designing learning opportunities for their classrooms (cf. Shulman, 1986; Kuntze, accepted), the
results related to research question (b) provide us with additional insight into potential professional
development needs of pre-service and in-service teachers. If we see modelling as a big idea for
mathematics, relevant for many content areas and for mathematical knowledge as a whole (e.g.
Martinez & Brizuela, 2009), the teachers’ views should be developed especially as far as connections
with a variety of content domains is concerned.

Going beyond, the findings call for empirical research into the role of the views examined here
for the teachers’ choice of learning opportunities in the classroom and into the structure of
professional teacher knowledge related to modelling. Such deepened analyses could open up ways of
effective professional development approaches.

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