Which personal factors affect mathematical modelling?
The effect of abilities, domain specific and cross domain-competences and beliefs on performance in mathematical modelling.

Christoph Mischo
Department of Psychology, University of Education, Freiburg, Germany
mischo@ph-freiburg.de

Katja Maass
Institute of Mathematics Education, University of Education, Freiburg, Germany
kaja.maass@ph-freiburg.de

Abstract
In this study, the effects of personal factors relevant for academic achievement and beliefs about mathematics on performance in modelling tasks are illuminated and empirically validated. Relations between performance in certain steps of the modelling process and reading competence, mathematical competence, general knowledge, word comprehension, fluid intelligence (reasoning ability) and beliefs about mathematics are particularized. Empirical validity of the postulated relations was tested by a structural equation approach with a sample of 959 students (grade six). Model fit and parameter estimates support the postulated relations between these personal factors and performance in mathematical modelling. Thereby, the construct of mathematical modelling was illuminated and conclusions for fostering mathematical modelling could be drawn by accounting for those personal factors affecting academic achievement that are related to mathematical modelling.

Keywords: mathematical modelling, competence, construct validation

1 Introduction

Mathematical modelling can be understood as solving realistic, open problems by means of mathematics. Teaching mathematical modelling at school is supposed to enable pupils to apply mathematics in their daily lives, thus giving them a better understanding of the world they live in and a better understanding of the utility of mathematics (Niss, Blum, & Galbraith, 2007, p. 6). Modelling has therefore been considered to be important for mathematics lessons among mathematics educators for decades (see e.g. Blum & Niss, 1991). As a consequence of the results of PISA, it became a part of the national German standards in 2004 (Kultusministerkonferenz, 2004). Carrying out a modelling process may be considered a challenge and requires not only mathematical competence, but a broader set of abilities and competences which affect academic achievement in general. Furthermore, performing in modelling tasks not only depends on the “skill” but also on the “will” of the performer. Hence, the role of beliefs fostering mathematical modelling should be taken into account.

The aim of the present study was to specify and to test empirically students’ personal factors (general abilities, cross-domain and domain-specific competences and beliefs about mathematics) which might affect their performance in modelling.

Within the discussion about mathematical modelling, most examples of modelling tasks as well as interventions, seem to focus on high or medium achieving students. (See e.g. the ICMI-Study volume Blum et al., 2007 or the ICTMA volumes, e.g. Haines et al., 2007). Furthermore, experience also shows that teachers tend to share the opinion that modelling seems to be too difficult for low achieving students. For this reason, this study will focus on the modelling performance of low achieving students. The study was carried out with students aged 11 to 12 at the German Hauptschule. This particular kind of German school is mainly visited by low-achieving students and by students from families with a low socio-economic status.

In the following, we will first outline our theoretical background, and based on this, state our hypotheses about which general competences might be of relevance for carrying out the modelling process.

2 Theoretical background
Mathematical modelling

In the literature on mathematical modelling, a variety of different definitions can be found (Kaiser & Sriraman, 2006, p. 303), as well as different assumptions about the modelling cycle (Maaß, 2004). We define mathematical modelling as the solving of a realistic problem by carrying out a so-called modelling process (Niss et al., 2007, p. 8). Based on Blum and Leiss (2005, p. 19), we conceptualize the following steps of the modelling process: (1) understanding the instruction and the real situation (situation model); (2) making assumptions and simplifying the situation model (real model); (3) mathematizing the real model (construction of a mathematical model); (4) working within the mathematical model (mathematical solution); (5) interpreting the solution; (6) validating the interpreted solution. The modelling process may be illustrated by the following scheme (Blum & Leiss, 2005).

The steps illustrated in the scheme have to be seen as an idealized scheme. This does not mean that every step postulated in the scheme corresponds to the actual cognitive processes of every individual or the way in which he or she chooses to solve the problem. As has been shown, students’ actual “modelling routes” (ways through the modelling process) can look very different (Borromeo-Ferri, 2007). Nevertheless, this scheme may illustrate different steps of an ideal modelling process.

Competences and modelling competence

In order to specify the term modelling competence, we will first take a look at general competences. There are various definitions for competences (e. g. Boehm, 2000, p. 309, Jank & Meyer, 1994, p.44, Baumert et al., 2001, p.141). These varieties are caused by different origins of the term from various branches of science and the distinction of certain types of competences. For this study, the following definition from the domain of pedagogy appears to be significant. Weinert (2001, pp. 27) defines competence as "the readily available or learnable cognitive abilities and skills which are needed for solving problems as well as the associated motivational, volitional and social capabilities and skills which are in turn necessary for successful and responsible problem solving in variable situations."

Accordingly, competences not only include abilities and skills, but also their reflected use in life and the willingness to put these skills and abilities into action. Within the discussion on mathematics education, Tanner and Jones (1995, p. 63) point out that motivation is an essential part of modelling competences: “Research has shown that knowledge alone is not sufficient for successful modelling: the student must also choose to use that knowledge, and to monitor the process being made.”
The exact understanding of *modelling competences* and skills is closely related to the definition of the modelling process. Different perspectives may therefore imply different views on modelling competences and skills. Basically, there has not yet been a comprehensive description of modelling competences (Blum & al., 2002, p. 271). Several studies refer to a number of sub-competences which are part of modelling competences. For example, Blomhøj & Jensen (2007) differentiate between the parts of the modelling process to be carried out, the mathematics to be used and the context in which students have to work.

The empirical study by Maaß (2006) produced evidence to suggest that:

1. students need competences to carry out the single steps of the modelling process
2. there are also further sub-competences which do not belong to a specific modelling step, but are needed throughout the whole modelling process.

The Maaß study yielded evidence for sub-competences based on a qualitative assessment of pupils’ written answers to modelling tasks. Based on the findings of Maaß, modelling competences can be regarded as a complex construct that contains a variety of sub-competences.

**Personal factors**

In educational research, different factors that affect learning outcomes have been identified (Wang, Haertel & Walberg, 1993; Helmke & Schrader, 2001). These main factors can be divided into

- (a) personal factors - individual determinants such as cognitive abilities (intelligence) or knowledge, motivational determinants, domain specific and cross-domain competences,
- (b) determinants of classroom quality and quantity (e.g. time on task, instructional clarity), and
- (c) determinants of the student’s family environment, such as parental help or parental stimulation (Helmke & Schrader, 2006, pp. 84). From all the factors listed above, intelligence and prior knowledge are the most relevant factors (Wang, Haertel & Walberg, 1993; Helmke & Schrader, 2001; 2006). In our study we will focus on personal factors.

Each step of the modelling process presumably requires skills which might be affected by personal factors. These personal factors comprise general cognitive abilities (intelligence), domain-specific and cross-domain competences, and beliefs.

We use the term *ability* to refer to human capacities which are relevant for task performance, which are long lasting and which are affected by heredity and experience (Desimone, Werner & Harris, 2002, p. 655). As Fleishman (1965, p. 138) pointed out, abilities may be considered to be “fairly stable traits, which in the adult, will not change very much unless the individual is subjected to some unusual environmental change”. In contrast to an ability, the development of a “competence” requires educational opportunities. Competences may also disappear over time if not used. In our study, we denote the term ability for *general cognitive abilities* (intelligence).

**Domain-specific competences** are required for tasks in an entire domain (e.g. mathematics). **cross-domain competences** are such competences which are relevant for task performance in different domains (e.g. reading competence). Task specific competences are relevant for certain tasks (e.g. modelling competence as relevant for modelling tasks).

In addition to the cognitive abilities, domain-specific (mathematical competence) and cross-domain competences, the performance in a modelling task is supposed to depend on motivational factors and beliefs about mathematics (Op’t Eynde, De Corte, & Verschaffel, 2002; Grigutsch, 1996 and Suthar & Tarmizi, 2010). Students’ beliefs about mathematics are the implicitly or explicitly held subjective conceptions students hold to be true about mathematics education, about themselves as mathematicians and about the mathematics class context. These in turn determine - in close interaction with each other and with students prior knowledge - their mathematical learning and problem solving in class (Op’t Eynde, et al., 2002). Grigutsch (1996) identified four categories of beliefs about mathematics: the aspect of scheme (mathematics is a fixed set of rules); the aspect of process (in mathematics, problems are solved); the aspect of formalism (mathematics is a logical and deductive science); and the aspect of application (mathematics is important for our lives and for society). Similar categories can be found within the international discussion: Ernest (1991) and Dionne (1984) differentiate between a traditional perspective, a formalist perspective and a constructivist perspective, which seem to correspond to the aspects of scheme, formalism and process.
The **aim of this study** is not to elaborate on the modelling task-specific components of the construct of modelling competences. Instead, we focus on more general personal factors which affect modelling performance. These personal factors comprise general cognitive abilities, cross-domain competences, domain-specific competences and beliefs about mathematics and learning mathematics. At first, we will set up a theoretical model which outlines these links. Then we will test empirically what Cronbach and Meehl (1955) in their seminal work about construct validation had termed the “nomological network” of the required abilities and cross-domain competences in mathematical modelling. That means that we try to link performance in mathematical modelling to the network of theoretical (but empirically supported) relationships of personal factors which affect academic achievement in general. Cronbach and Meehl (1955) characterized the establishing or expanding of a theoretical network of a construct as “learning more about” a particular construct.

**3 Links between mathematical modelling, abilities, domain specific and cross-domain competences**

**General considerations**

In the following, we differentiate between **modelling competence** and **performance** in mathematical modelling. “Modelling competence” refers to the modelling tasks’ specific competences which enables and motivates an individual to solve modelling tasks. Competences are not observable but may (or may not) be indicated by performance scores. For several reasons, however, it may happen that an individual demonstrates a low performance which does not necessarily imply a low competence (e.g. in the case of a badly constructed test; e.g. Sophian, 1997; Wood & Power, 1987). Moreover, the term “competence” implies a “surplus meaning” (Cronbach & Meehl, 1955) which means that the construct of competence is theoretically “richer” than the pure observable performance and contains assumptions about the “deep structure” (Wood & Power, 1987) of the performance. If there are no other empirical indicators for a competence, however, than the ones in a given task, it cannot be differentiated between competence and performance on an empirical level. As our modelling tasks are newly constructed and as we do not have any other indicators for modelling competence than the ones of our tasks, we use the term “performance” and thereby try to avoid the risk of a tautological equalization of performance and competence on an empirical level.

In order to specify relationships between personal factors and performance in each step of the modelling process, **how modelling competence will be operationalized and assessed** is crucial. Modelling competence can be assessed using open answer or multiple choice formats (Haines, Crouch, & Davies, 2001). Answering in multiple choice format requires e.g. reading questions and distractor items for each step of the modelling process. Answering in open answer format requires writing competence and a high motivation to write down the solution and the way it was reached. Therefore, the relationships between modelling competence and general competences may depend on the answer format. Hence, different sets of relationships should be specified for (a) modelling competence assessed in multiple choice answer format and (b) modelling competence assessed in open answer format.

In our study, however, the open answer format turned out to be problematic with regard to reliability (very low Cronbach’s alpha). Therefore, the open answer format scale was not used for further analyses and will not be presented in the following. For reasons of clarity, we specify relations only for modelling competence assessed in multiple choice answer format. The following postulated relationships between general abilities and cross-domain competences and steps of the modelling process do not mean that these competences are the only ones that are required in order to solve the according step of the modelling process. In fact, the general academic competences outlined above can be considered as relevant factors which affect the solution of the corresponding step of the modelling process. We do not postulate that these general academic abilities and cross-domain competences are the only ones which have an effect on competence in mathematical modelling. Rather, we wanted to investigate how personal factors (including beliefs about mathematics) that are relevant for learning mathematics and learning at school in general, also affect competence in mathematical modelling.
In the following, we will outline which personal factors (general abilities, cross-domain and domain-specific competences and beliefs about mathematics) may affect all steps of the modelling process or the individual steps.

**Personal factors affecting all steps of the modelling process**

If modelling tasks are presented in a written format, students should be able to read and understand written information. In the PISA study, reading competence is understood as the ability to understand texts in terms of the content, intentions and formal structure (Baumert et al., 2001). Therefore reading competence (cross-domain competence) is supposed to affect all steps of the modelling process, particularly in a multiple choice answer format where questions and distractor items are presented for each step of the modelling process.

As general intelligence (general ability) is one of the most powerful predictors of school performance (Helmke & Schrader, 2006; Wang, Walberg & Haertel, 1993), general intelligence is supposed to be related to all steps of the modelling process. General intelligence is usually operationalized by tests of fluid intelligence in the sense of Cattell (1971) which comprises nonverbal logical reasoning.

In relation to beliefs, students with traditional and rigid beliefs about mathematics (as described by the aspect of scheme, see above) are probably less motivated to solve a modelling task than students holding beliefs that mathematics is a useful tool in order to solve real world problems.

**Personal factors affecting performance in certain steps of the modelling process**

In order to illustrate our reflections on these competences, we will look at two task examples.

Task Water Saving

“It is a well-known fact that if you let the water run while brushing your teeth, a family of four wastes 26,000 litres of water per year.” This quote from a newspaper article says that every family can save 26,000 litres of water every year if they turn off the tap when brushing their teeth. What do you think about that? Is it really possible? Give reasons!

Task Time at school

Ingo thinks that he spends too much time in school. “Most of the year I am at school,” he says. What do you think about this?

We will now look at the different steps of the modelling process and the general competences which may be required to carry out the particular step.

**Step (1) Understanding the instruction and the situation presented in the modelling task**

Besides general intelligence (general ability) and reading competence (cross-domain competence), step one of the modelling process may require the ability to not only to read the question but to understand words and expressions mentioned in the question or the distractor items. Word comprehension belongs to so called *crystallized intelligence (general ability)* in the sense of Cattell (1971), which means culture specific knowledge about facts and semantics.

**Step (2) Constructing a real model**

In modelling tasks, all necessary information is often not given in the text, but has to be inferred from general knowledge (general ability). Here, students have to simplify the situation presented in the tasks, estimate missing information from their general knowledge and complement information from their general knowledge in order to solve the task. In the water-saving task, for example, students may simplify the situation by assuming that the brushing of teeth always takes exactly the same time. They have to make assumptions, for example that the average family consists of about four members, that brushing teeth always takes about three minutes etc. and that within three minutes, 0.5 litres of water runs down the drain. In the second, task they may assume that on an average school day, Ingo spends six hours at school, etc. The ability to make these assumptions may
be related to **general knowledge**. Therefore we assume that general knowledge is relevant in order to solve step two of the modelling process.

**Step (3) Setting up the mathematical model**

In this step, the real model has to be transformed into a mathematical model consisting of mathematical variables and numerical relationships. In relation to the relatively simple tasks given above, this means that the assumptions as described have to be transferred into a calculation. E.g. pupils have to calculate how many weeks they normally spend at school (52 weeks minus e.g. 13 weeks of school holidays), that a school week has five days and each school day has six hours, resulting in \((52-13) \times 5 \times 6\). A similar calculation has to be set up for the water saving task. Hence, this requires **mathematical competence (domain specific competence)** in the sense of the ability to solve calculation tasks and word problem tasks.

**Step (4) Working within the mathematical model**

Within this step, a term like the one mentioned above has to be calculated. **Mathematical competence** is required in order to work mathematically within the mathematical model and to solve the task at hand.

**Step (5) Interpreting the mathematical solution**

If pupils obtain e.g. 20,000.00 litres as a result in the saving-water task, they have to explain what this means. In the other task, pupils may end up with e.g. 14,000. Thus, the mathematical solution within the mathematical model has to be transformed into semantically meaningful terms. In the first case, this means that 20,000 litres of water can be saved, in the second this means that Ingo so far has spent 14,000 hours at school. In order to solve this step, students have to apply their individual thesaurus of words and transform the semantic meanings of terms to mathematical results, which requires mathematical competence as well.

**Step (6) Validating the interpreted solution**

The semantically interpreted solution has to be validated. This means that pupils have to reflect on their calculation critically. Sometimes there is also the possibility to compare the values with other values that are familiar in order to see if the range of values is appropriate. Students have to check the plausibility of the interpreted solution considering their general knowledge. Therefore, **general knowledge** is supposed to enhance solving this step of the modelling process.

We are quite aware of the fact, that metacognitive and motivational competences and competences in reasoning mathematically may be required to solve modelling tasks (see also Maaß, 2006; see discussion section).

### 4 Hypotheses

Instead of formulating single hypotheses for each step of the modelling process, we give an overview of the postulated relationships in Table 1 (following page).

### 5 Methods

**Data collection and sample**

All data was assessed with questionnaires in October 2008 in 54 classes in the local area of Freiburg, Germany (N = 959). Students were in grade six of the German *Hauptschule*, as in this grade, mathematical modelling is an integral part of the curriculum. In our sample, 58.5 percent were boys (n = 560) and 41.5 percent were girls (n = 398). One questionnaire was incomplete and therefore not included in the study. The average age of the students was 11.55 (standard deviation = 0.68). About 69 % of the students indicated that they spoke German at home, 10 % indicated another language, and 21 % said they spoke German and another language.
Step 1: Setting up a situation model:
Comprehension of the task, the text and the instruction

Step 2: Setting up a real model: construction of mental representations, connecting modelling task and general knowledge

Step 3: Setting up a mathematical model
Mathematical competence

Step 4: Finding a solution within the mathematical model
Mathematical competence

Step 5: Interpreting the mathematical solution
Mathematical competence, word comprehension

Step 6: Validation
General knowledge

<table>
<thead>
<tr>
<th>All steps</th>
<th>Reading competence, fluid intelligence, beliefs (motivation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1: Setting up a situation model:</td>
<td>Word comprehension</td>
</tr>
<tr>
<td>Comprehension of the task, the text and the instruction</td>
<td></td>
</tr>
<tr>
<td>Step 2: Setting up a real model: construction of mental representations, connecting modelling task and general knowledge</td>
<td>General knowledge</td>
</tr>
<tr>
<td>Step 3: Setting up a mathematical model</td>
<td>Mathematical competence</td>
</tr>
<tr>
<td>Step 4: Finding a solution within the mathematical model</td>
<td>Mathematical competence</td>
</tr>
<tr>
<td>Step 5: Interpreting the mathematical solution</td>
<td>Mathematical competence, word comprehension</td>
</tr>
<tr>
<td>Step 6: Validation</td>
<td>General knowledge</td>
</tr>
</tbody>
</table>

Table 1- Steps of the modelling process assessed in multiple choice format and postulated relationships to personal factors

*In this study, we focused on cognitive abilities and competences and did not consider motivation. Theoretically, however, motivation is supposed to affect performance at all steps of the modelling process.

Assessment of variables

Performance in modelling

As no appropriate test for assessing modelling competences of the target group exists, we developed a modelling test with open and closed-format items (for details see Maaß & Mischo, 2011). As assessment in open answer format was not reliable, only assessment in multiple choice answer format is reported. The multiple choice format contained one item for each of the six steps of the modelling process (see Fig. 2), and for each item there were five answer options. From these answer options, the correct one(s) had to be ticked as “correct”. Following Bortz and Doering (Bortz & Doering, 2002, p. 214), for each of the five answer options, one point was given if it was answered correctly (ticked if correct and not ticked if false). No points were given if the pupil answered incorrectly. In the following, an example is given for the multiple choice items for the first step (understanding the instruction) and the second step (simplifying the situation model). Regarding the first step, we presented four incorrect distractor items (the third answer option is correct). In the second step, we presented four potentially relevant assumptions and one non-relevant assumption (third answer option) in order to construct the real model.

Christopher loves the three summer months June, July and August. When the weather is fine, he goes to a small lake as often as possible. “Then I’ll spend 1000 hours at the lake,” he says.

Can this be right? Please do not try to solve the task right now, but first read the following carefully and tick the answers that are correct.

1) What are you supposed to do in the task?
To find out …

[ ] … how many months the summer has.
[ ] … what Christopher does when he is not at the lake.
[ ] … how many hours Christopher will spend at the lake.
[ ] … why Christopher goes to the lake.
[ ] … how many months one year has.
The answer seems to be quite obvious, however, as the task was designed for low achieving students aged 11 to 12 who often have an immigration background, it seemed quite necessary to assess step 1 (understanding the situation).

2) Which considerations may be relevant to solving the task?
   - Christopher lives near the lake.
   - It rains about 10 days in a month.
   - Five boats are sailing on the lake.
   - Christopher spends about four hours a day at the lake.
   - One month has about 30 days.

Every student worked on four tasks with four different contents. To control the effect of the task’s content and order, the sequence of tasks was balanced according to a sequentially balanced Latin square.

Reading competence
   Reading competence was assessed using a standardized German reading comprehension test for grades one to six (ELFE 1-6; Lenhard & Schneider, 2006). In the present study, we applied subscales for reading comprehension on sentence level and text level.

Fluid intelligence
   Fluid intelligence was operationalized by using the scale “reasoning ability on matrix tasks” from a German Version of Cattell’s Culture Fair Test (Weiss, 2006).

Word Comprehension
   This aspect of crystallized intelligence was assessed by applying the scale “word comprehension” from the intelligence scale mentioned above (Weiss, 2006).

Mathematical competence
   Mathematical competence was assessed by using a standardized mathematics test (the German Mathematics Test DEMAT 4+; Goelitz, Roick, & Hasselhorn, 2006). This test consists of several types of tasks: quantity comparisons tasks, number line tasks, word problems and addition and multiplication tasks. This test is designed for students aged 10 – 11, but comprises all mathematics aspects needed for solving the modelling tasks used in the study. (Because modelling is supposed to be very demanding for students, the mathematical demand should not be too high).

General knowledge
   This ability was measured by a written adaption of the subscale “general knowledge” from a German version of the Wechsler Intelligence Scale for Children (oral test, Hamburg Wechsler Intelligence Scale for Children; Tewes, Rossmann & Schallberger, 1999). The test needed to be adapted because it is usually used for individual oral interviews. One example item is: “Who was Christopher Columbus?” According to the test manuals, all reliabilities of these standardized scales were higher than .70.

Beliefs
   In order to assess beliefs about mathematics, some scales from the PISA-study were adapted (Ramm et al., 2006). Because these scales were adapted and modified, reliabilities (Cronbach’s alpha) were calculated from the sample data. We used the following scales:
   - Mathematics as application of rigid schemas (five items, Cronbach’s alpha in the sample = .72, example item = “In mathematics there exists only one way to solve a task” rating item from 1 = “does not apply” to 4 = “applies”)
   - Utility of mathematics (four items, Cronbach’s alpha in the sample = .66, example item: “What we learn in mathematics at school helps me to solve real world problems”, rating item from 1 = “does not apply” to 4 = “applies”).
Mathematics as a self-discovering process (three items, Cronbach’s alpha in the sample = .65, example item = “In order to solve a task, one’s own ideas are required”, rating item from 1 = “does not apply” to 4 = “applies”). The reliabilities of the latent constructs (composite reliabilities according to Bacon, Sauer & Young, 1995) are reported in the results section below.

6 Results

A structural equation model was constructed representing the relationships postulated in Table 1. On the left side of the model graphic, general competences and beliefs were specified as exogenous latent constructs (exogenous: variables which cause other variables in a model). In the case of different scales indicating a latent construct (e.g. for reading competence and mathematical competence), all scales were used as manifest indicator variables. In the case of only one scale indicating the latent construct, the scale was divided into an odd-item subscale and an even-item subscale (for the constructs knowledge, word comprehension and fluid intelligence). Each subscale served as a manifest indicator variable.

On the right side of the model graphic, the endogenous latent construct of mathematical modelling competence (assessed in multiple choice format) was specified by the manifest multiple choice-scores for each step of the modelling process (endogenous: variables which are caused by other variables in a model). Correlations between the exogenous latent variables were permitted.

In order to test the central research question (relationships between general competences and mathematical modelling), two structural equation models were constructed. In the first model (independent model), all paths between exogenous variables and endogenous variables (mathematical modelling competence) were fixed to zero. These paths were:

- from reading competence to modelling competence (readcomp_modcomp in figure 2);
- from mathematical competence to the steps mathematizing, calculating and interpreting the solution (mathcomp_step3, mathcomp_step4 mathcomp_step5);
- from general knowledge to setting up a real model (knowledge_step2) and to interpretation (knowledge_step5) and validation (knowledge_step6);
- from word comprehension to understanding the instruction and to the interpretation of the solution (wordcom_step1, wordcom_step5);
- and from fluid intelligence to latent modelling competence (fluid_modcomp).

Furthermore, paths from the beliefs to latent modelling competence were set to zero.

In the second model (relationship model), all these paths were estimated without restrictions. The theoretical model is depicted in Figure 2 (following page).

All analyses were carried out with SPSS AMOS Version 19. The model fit of the independence model was good, but the fit of the relationship model was slightly better (see Table 2).

<table>
<thead>
<tr>
<th>Model</th>
<th>CMIN</th>
<th>DF</th>
<th>P</th>
<th>CMIN/DF</th>
<th>CFI</th>
<th>RMSEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independence model</td>
<td>533.94</td>
<td>198</td>
<td>.000</td>
<td>2.69</td>
<td>.922</td>
<td>.042</td>
</tr>
<tr>
<td>Relationship model</td>
<td>393.32</td>
<td>187</td>
<td>.000</td>
<td>2.10</td>
<td>.952</td>
<td>.034</td>
</tr>
</tbody>
</table>

Table 2: Theoretical model: fit of the independence model and the relationship model

Note. Independence model: paths between exogenous and endogenous variables fixed to zero, relationship model: freely estimated paths between exogenous and endogenous variables. CMIN = minimum discrepancy, DF degrees of freedom, CMIN/DF (values less than 3 indicate good fit of the model to the data), CFI = comparative fit index (values greater than .90 indicate good fit), RMSEA = root mean square error of approximation (values about .05 or less indicate good model fit).
Figure 2 - Theoretical model

Note. Independence model: readcomp_modcomp=0, mathcomp_step3=0, mathcomp_step4=0, mathcomp_step5=0, knowledge_step1=0, knowledge_step2=0, knowledge_step6=0, wordcom_step1=0, wordcom_step5=0, fluidint_modcomp=0, beliefrigid_modcomp=0, beliefutility_modcomp=0, beliefdiscovery_modcomp=0, e1 - e22 = error variances, d1 = disturbance variance (latent variable’s error variance).

In order to test the difference in model fit of the independence model and the relationship model, a chi-square difference test was performed (see Table 3).

<table>
<thead>
<tr>
<th>Model</th>
<th>CMIN</th>
<th>DF</th>
<th>CMIN/DF</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independence model</td>
<td>11</td>
<td>140.64</td>
<td>12.72</td>
<td>.000</td>
</tr>
</tbody>
</table>

Table 3 - Comparison of theoretical models: difference in fit of the theoretical independence model assuming relationship model to be correct

Note. Independence model: paths between exogenous and endogenous variables fixed to zero, relationship model: freely estimated paths between exogenous and endogenous variables. CMIN minimum discrepancy, DF degrees of freedom, CMIN/DF (values less than 3 indicate no relevant difference of fit), p-values lower than .05 indicate significant difference in model fit.
The chi-square difference test yielded a significant difference in the goodness of fit of the independence model as compared to the relationship model. Despite the fact that the model fit of the independence model was good, the model fit of the relationship model was even better. In other words: The parameter estimates between exogenous variables (general competences and beliefs) and the endogenous variable (mathematical modelling competence) were estimated freely (and not restricted to zero). This in turn resulted in a better fit of the structural equation model to the data.

Despite the good model fit of the relationship model, the following paths were not statistically significant: from general knowledge to the validation of the solution (step six); from utility belief to modelling competence; from belief “mathematics as a self-discovering process” to modelling competence; and the factor “loading from modelling competence to step 4 calculating”. For reasons of parsimony, these non significant paths were deleted from the model.

The overall fit of the final model with significant paths only is summarized in Table 4.

<table>
<thead>
<tr>
<th>Model</th>
<th>CMIN</th>
<th>DF</th>
<th>P</th>
<th>CMIN/DF</th>
<th>CFI</th>
<th>RMSEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relationship model</td>
<td>365.61</td>
<td>171</td>
<td>.000</td>
<td>2.14</td>
<td>.956</td>
<td>.035</td>
</tr>
</tbody>
</table>

Table 4 – Final model fit (only significant paths)

Note. CMIN minimum discrepancy, DF degrees of freedom, CMIN/DF (values less than 3 indicate good fit of the model to the data), CFI = comparative fit index (values greater than .90 indicate good fit), RMSEA = root mean square error of approximation (values about .05 or less indicate good model fit).

The final model including only significant paths with standardized parameter estimates as illustrated in Figure 3 (following page).

All algebraic signs of the path coefficients were as expected: all path coefficients being positive except for the expected negative path from mathematics beliefs (rigid schemas) to modelling competence. The composite reliabilities of the latent constructs (Bacon, Sauer & Young, 1995) in the final model were: reading competence (readcomp) = .78, mathematical competence (mathcomp) = .50, general knowledge (knowledge) = .49, word comprehension (wordcom) = .84, fluid intelligence (fluidint) = .77, and modelling competence (modcomp) = .63. As all path coefficients in the final model are significant, most of the theoretical assumptions about an effect of general competences on performance in certain steps in the modelling process, as well as on modelling competence in general, could be confirmed.

7 Summary and Discussion

Summary

The main research question of the present paper was: Which personal factors (general abilities, cross-domain and domain specific competences and beliefs about mathematics and mathematics education) are required to solve mathematical modelling tasks? Based on a scheme consisting of six steps in an ideal modelling process, relationships between general competences and performance in each step of the modelling process were specified as a structural equation model (relationship model). Contrasting to a structural equation model including paths from personal factors to mathematical modelling, an alternative structural equation model was specified in which all paths from exogenous to endogenous variables were fixed to zero (independence model).

The fit indices of both models suggested a good fit of both structural equation models, but also an even better fit of the relationship model compared to the independence model. After eliminating non-significant paths, the model gives information about the relationships between personal factors and performance in mathematical modelling.
In solving modelling tasks, the first step (understanding the instruction) is facilitated if students show higher scores in word comprehension. The second step of the modelling process (simplifying the situational model) is affected by general knowledge, whereas mathematical competence had an effect on mathematizing the real model (third step), calculating (fourth step) and interpreting the mathematical solution (fifth step). The semantic interpretation of the mathematical solution is also affected by word comprehension ability.

According to our prior assumptions, modelling competence in general (assessed in multiple choice format), could be predicted by reading competence, fluid intelligence, utility beliefs of mathematics and by lower values in beliefs that mathematics means applying rigid schemas.

In a nutshell, performance in different steps of the modelling process could be predicted by general competences in a theoretically plausible manner. Thereby the nomological network, i.e. the network of theoretical relationships between mathematical modelling and other constructs, could be particularized.

Despite this general conclusion, some of the results did not confirm to the assumptions. The fourth step “calculating” did not relate to the latent construct of modelling competence (no significant factor loading). An explanation for this could be that working within a mathematical model (calculating) is not specific to modelling competence, and therefore does not relate to the other steps of modelling performance.
Furthermore, the validation of the modelling task’s solution (step six) could not be predicted by general knowledge (no significant path coefficient). However, as reading competence, fluid intelligence and beliefs predicted general modelling competence, an indirect effect of these exogenous variables remains on the validation of the solution (step six). Lastly, two of the three beliefs assessed in the study had an effect on the modelling competence (negative effect of the belief “mathematics means applying rigid schemas” and positive effect of utility-belief of mathematics), whereas an effect of belief “mathematics as a self-discovering process” could not be confirmed. The results of the path diagram are summarized in Table 5.

<table>
<thead>
<tr>
<th>Steps in the modelling process</th>
<th>Postulated relationship to personal factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>All steps</td>
<td>Reading competence, fluid intelligence, beliefs</td>
</tr>
<tr>
<td>Step 1: Setting up a situation model: Comprehension of the task, the text and the instruction</td>
<td>Word comprehension</td>
</tr>
<tr>
<td>Step 2: Setting up a real model: construction of mental representations, connecting modelling task and general knowledge</td>
<td>General knowledge</td>
</tr>
<tr>
<td>Step 3: Setting up a mathematical model</td>
<td>Mathematical competence</td>
</tr>
<tr>
<td>Step 4: Finding a solution within the mathematical model</td>
<td>Mathematical competence</td>
</tr>
<tr>
<td>Step 5: Interpreting the mathematical solution</td>
<td>Mathematical competence, word comprehension</td>
</tr>
<tr>
<td>Step 6: Validation</td>
<td></td>
</tr>
</tbody>
</table>

Table 5 - Empirical relationships between performance in different steps of the modelling process and personal factors

Methodological discussion

For some of the latent constructs (e.g. for the latent variable of modelling performance), the composite reliability was almost mediocre. In fact, this is a dilemma which has been well known as the “bandwidth-fidelity dilemma” in psychometrics since the seminal work of Cronbach (1961). Broader constructs (with greater bandwidth), such as the latent variable of modelling performance, cannot be measured very reliably (low fidelity). Regarding the direction of the paths, it seems plausible that general competences (such as reading competence or fluid intelligence) affect modelling performance and not vice versa. With respect to beliefs, however, a converse effect seems equally plausible: Beliefs about mathematics can also be the effect of modelling competence.

Contribution to mathematics education

This study sheds light on the question of what personal factors are needed to carry out modelling processes. We can see that general intelligence (fluid intelligence), reading competence and beliefs affect all steps of the modelling process by being linked to the latent construct of modelling competence. Basic mathematical competence, word comprehension and general knowledge are important personal factors needed for carrying out certain steps of a modelling process. Thus, this study supplements the previous study by Maaß (2006) as it systematically looks at which personal factors are needed for carrying out a certain step in the modelling cycle. However, the reported results were obtained with low achieving students aged 11 to 12 in a multiple choice format, and we did not
explicitly look at metacognitive, motivational and mathematical reasoning competences. Therefore, we cannot draw any conclusion about these aspects.

From the point of mathematics education, these results should provide indications for fostering the competence in mathematical modelling: general knowledge, reading competence, word comprehension and mathematical competence in general are relevant personal factors that are also required for mathematical modelling. These factors can and should be enhanced by school education, be it in other subjects in preparation for modelling, or in mathematics education through modelling.

Whereas the ability of fluid intelligence can hardly be promoted at school, the students’ beliefs about mathematics can be affected by mathematics education - and may indeed be the effect of mathematics education itself. Helping students to develop beliefs that mathematics is not the application of rigid schemas may also be helpful in promoting modelling competence. This can be done by student-centred mathematics lessons, which involve students in inquiry, motivate them to search for their own solution strategies and engage them in solving realistic and complex problems from their daily and/or professional lives.

It was, however, surprising that some of the results did not confirm the assumptions. For example, the fourth step “calculating” did not relate to the latent construct of modelling competence and the validation of the modelling task’s solution (step six) could not be predicted by general knowledge (no significant path coefficient). Further, as the results also show, an effect of the belief “mathematics as a self-discovering process” could not be confirmed. Further research, e.g. in qualitative analyses, should provide an in-depth look at these aspects.

As these findings resulted from a correlational study, their validity should be replicated in interventional and experimental studies. Further studies should also be carried out with students in other grades. Finally, this study was based on a multiple choice answer format which may have had an impact on the results. For this reason, further studies which are based on different question formats should be conducted.

References


Kultusministerkonferenz (2004). Bildungsstandards im Fach Mathematik für den Mittleren Schulabschluss [educational standards in the subject of mathematics].


