Mathematical Modeling by identification: a case study in the laboratory of control applied to the identification of a servo mechanism

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Abstract
The mathematical modeling of physical systems takes two forms. The first form is by differential equations. The second form is by identification methods that may be linear or nonlinear. The mathematical modeling of physical systems for identification has been widely discussed by academia, but most of works are developed within the simulation and not experimentation. Experiments conducted to real physical systems are scarce in the literature and a scarcity. The design of controllers is needed for process conditions regulatory and servant and is subject to the characteristics of the plant to be controlled. However, an actual process has aspects that increase the complexity of this project such as: delay of transport, noise stochastic fluctuations, the dynamics change in time. It is presented in this work, the mathematical modeling identification of a mechanical system fragile, and its dynamic changes over time, by the least squares non-recursive, so that the controller design is best suited to dynamic characteristics of this process.

Keywords: Mathematical Modeling, least squares, Engineering Education.

1. Introduction

The development of educational tools for institutions education does reflect the basic pillars of education as not just do it, but knowing how to do and redo. From this definition, as a process of formation of human competence, one can trace clearly the relationship between theory and practice. The school now aims to preparing people to cope with the rapid changes of world (Santana, 1995). People who should learn to learn and thus become able to continue always learning. Thus the curriculum of today should not promote both learning content and specific skills, but favorable conditions for the implementation and integration of this knowledge.

This is feasible through the proposition of situations to encourage the development of the student's ability to problem solving, many of them common in their day to day. In this respect, the research lab needs to be fostered as attitude every day, not just as special activity and special structure (Souza, 2000). Therefore, development of human competence, the key issue is to make the normal laboratory learning environment, making the practices frequently, combining well, simplicity and modernity. The relationship between theory and practice requires a constant renewal of teaching tools in the laboratory (Demo, 1998). However, a better compromise with the school requires a trial better
and more frequently in conducting experiments, this would lead to the need for large infrastructure material support. This is problematic in many cases, difficult to put into practice because of the scarcity of resources. Stands a pedagogical problem underlying the central control in education: establish the weighting right and appropriate interdependence between the base and conceptual experimentation. This issue, inherent the actual engineering education (Feisel, 2005), assumes contours highlighted when the object in question is the education control, they may be marked twofold clear: first is physical - when applied to control, control is always something material, real - while the second is purely abstract, as basic concepts of control, such as stability and feedback, are strictly mathematical. Emerges, this duality, a situation assumes known contours distinction between theory and practice, becoming inherent in nature control (Kheir, 1996). Therefore, it is noted that the practices and laboratory experiments in control education beyond the character of an activity complementary to learning, taking the leading role - and extremely prominent - an equilibrium solution enabling appropriate balance between theory and practice. And more: the use of continued laboratory practices associated with development of new laboratories and control environments computer aided - and their integration into the structure curriculum - is considered an essential element of education in control and is an indispensable tool for the establishment of its new paradigms (Antsaklis, 1998).

It is known that the higher the number of senses involved one study, the higher the setting, the seizure and the ability to perception of students. Thus, for example, the assimilation of a concept arising only from a reading of a text is well less than the recovery seen whether this reading is accompanied by auditory phenomena, concomitant with preview images related to the same concept (Gomes, 2006).

From these questions, it became necessary to develop an interfacing system that allows data communication Matlab with a kit of teaching control servo-mechanism, developed by Datapool, modernizing discipline of Automatic Control Lab, taught in CEFET-MG 1. The proposal aims to motivate students to take control kit using Matlab, thereby fostering the consolidation of knowledge acquired in theoretical basis. The question therefore is developing a system for transferring data to the control and automation, computer, low cost and components easily obtainable in the market. Computer control, presented Figure 1 is a relatively new area of research. The advantages to replace analogue controllers by digital in terms of flexibility and cost were already glimpsed in the decade 50, and until the 60s, applications were restricted (Hemerly, 2000). From the 70 developing computers enabled the improvement of product quality, as well as reliability and flexibility, thus contributing to increasing productivity.

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Figure 1. Control System with Computer

Currently, a potential user of computer control has hugely vast resources of software and hardware. Nevertheless, this diversity of resources does not impact educational kits tools have worked in terms of theory as a place roots, frequency response, stability, measures performance. However, as we have software with all resources worked in the theoretical realm, such as Matlab, becomes the
main orientation of this work to develop a means allowing applying these tools worked theoretically in terms of system identification in a real device like the kit servo Datapool.

The central problem here is that the kit is only available in the laboratory for use in teaching the standard techniques of control of dynamic systems. However, this limits you to a phenomenological model presented by the equipment manufacturer. It was envisioned by this research team to develop a teaching tool that would facilitate the learning also of modeling techniques for systems identification.

The work is structured as follows: first presents itself, in section 2, a formalization of modeling and identification systems, explaining why to use it, immediately, in section 3, mathematicize the method of least-squares, tracking is explained as is done communicating with Matlab, the results are presented in Section 5. Section 6 concludes this work.

2. Modeling and System identification

Means, for modeling and identification, to determine the mathematical model of a system, representing the essential aspects appropriately is use such as diagnosis, monitoring, optimization and control. The formula used to arrive at a mathematical model of a system can be verified in Figure below.

![Figure 2. Procedures for developing a mathematical model of the process being monitored](image)

The design of industrial controllers is complex and in some cases does not lead to the control loop to a desired performance. But for some processes to be controlled, has achieved a satisfactory performance by using design techniques enshrined. The projects use techniques of approximate dynamic models of the processes to be controlled as well as evaluating the performance of knitwear industry based on the concept of performance indicators. The ideal design for controllers can be accomplished when we know the characteristics of controlled processes. Therefore, the adjustment of controller parameters should be based on the characteristics of the process being controlled. The transfer function of the process to be controlled, however, is not always available or known. The general procedure in these cases is the use of approximate mathematical models, which have representative parameters, significant for the dynamics of the system and allow the design of controllers.

Usually, to model a real process, one can apply techniques in open loop or closed loop. To apply the techniques of identification, apply signals on its input, open loop, or using closed loop techniques, as the method of relay (Brosilow, 2002). This allows us to measure parameters such as gain and critical period, time constants and time delay, among others. These procedures, used by most other methods for tuning controllers, based on parameters of reduced-order models represent dynamic systems that allow higher-order, making the procedures simpler and tuning feasible. The dynamics of a process can be determined by the response to disturbances such as pulse, step, ramp, or other signs deterministic. These models have about the process dynamics, are important and sufficient for determining the tuning parameters of the controller. The identification of systems and processes, there are established techniques and empirical methods such as the reaction curve, the method of Chen
method period and gain critical (Carmo, 2006). There are also the most recent methods for the representation of the model of a process; the method of relay is example (Coelho, 2004). Although established methods, restricted to the representation for models of dynamic processes that have a first or second order which tolerates only delay shipping. Some of these methods are academics as the gain and critical period, where their application is, in some cases, dangerous. Also these methods do not allow the representation of models for systems that are more real, especially when they are inherent transport delay, oscillations, and noises.

The model is more inadequate when the process is associated with other characteristics inherent to mechanical systems such as loosening, wear or friction. Here the adaptative control or predictive (Almeida, 2002) these systems becomes very attractive alternative because it takes into account more realistic models, but increasing the complexity of the implementation of control. For a thinner design of the controller, we use techniques that allow the identification of a more accurate model, such as recursive least squares estimators and non-recursive, the stochastic approximation algorithm, instrumental variable algorithm, instrumental variable estimation or symmetric matrix extended (Aguirre, 2000). Another issue that must be taken into consideration is that real processes running show behaviors which are not found in simulation environments and certain means of identification, such as the reaction curve, become inadequate or lead to erroneous models of the process control, in the laboratories of CEFET-MG, the discipline of Automatic Control Laboratory II, apply the concepts acquired in control theory for controlling industrial plants. Among the equipment used can highlight a small plant for the speed control or positioning (Figure 3).

![Figure 3. Plant for speed control and positioning](image)

The equipment manufacturer has submitted a transfer function of the equipment and software to control and that the controller is already designed. But the software does not allow the student to perform all the engineering process, from design to implementation, because the controller has already projected digitally. Is associated with this problem, the fact that the mechanical parts of the plant are fragile and already in its five years of operation have suffered wear and depreciation, thereby changing the dynamics of the system by modifying the transfer function given by the manufacturer. Therefore, the software made by the manufacturer became unusable, and any project undertaken under the transfer function adopted. Therefore, it had a big problem: functional equipment, but which, however, could not be used by modifying its transfer function.

Then, emerge from this problem, a solution by identifying the system, aiming to estimate the new transfer function, coming thus to model the process. There were two ways of estimating the process model, one using traditional techniques such as reaction curve or relay, which he found great difficulties because of the characteristics found in practice such as noise on the control loop, and another way was to use techniques for working with more real situations as the method of least squares estimator of the non-recursive, and the latter to be adopted.
3. The least squares estimator

The core of the development of methods for identification of systems comes from the least squares estimator (Ljung, 1999). The structure of the identified model, the noise in the mesh, as well as resolution of information that is contained in the data influence the accuracy of the estimation. You can make estimates of the type online or off-line. However, estimation of model parameters is considered, in principle, the order of the model, and that samples input and output are available in every interaction. The principle of least squares was made in the late 18th century to predict the trajectory of planets and comets from observations by Karl Gauss Fiedrich. K. F. Gauss found that the unknown parameters of the mathematical model should be selected so that the most likely value of unknown quantities is the one that minimizes the sum of the squares of the difference between the values currently observed and the values multiplied by numbers that measure the degree of precision where the more accurate the measure, the greater its weighting (Coelho, 1999).

3.1 Estimator of the non-recursive least squares

Every physical process can be characterized by a differential equation coming from the phenomenological modeling or modeling for identification (Geromel, 2004). However, the most common way of representing a system is the use of frequency representation, in the frequency domain. This representation, when expressing the ratio of output to the input has a transfer function in frequency domain. From the computational point of view, it is more important to represent a transfer function, representing a process in the frequency domain by a complex process in the discrete complex frequency domain. To represent a discrete transfer function, consider a linear physical process characterized by an input $u(t)$, output $y(t)$, for a disturbance $e(t)$, and with a transfer function of the form (Eq. 1):

$$A(z^{-1})Y(z) = z^{-d}B(z^{-1})U(z) + E(z)$$

(1)

Where:

$$A(z^{-1}) = 1 + a_1 z^{-1} + ... + a_{na} z^{-na}$$

(2)

$$B(z^{-1}) = b_0 + b_1 z^{-1} + ... + b_{nb} z^{-nb}$$

(3)

We can represent the transfer function by a discrete difference equation, thus enabling the simulation:

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) - ...$$
$$-a_{na} y(k-na) + ...$$
$$... + b_0 u(k-d) + b_1 u(k-d-1) + ...$$
$$+ b_{nb} u(k-d-nb) + e(k)$$

(4)

Of Eq.3 can be seen that has the $na+nb+1$ parameters to estimate, to determine $a_i$ e $b_j$, should be used for entry and exit process; the term $e(k)$ represents the modeling error, measurement error or noise in the output type of stochastic or deterministic. You can re-write Eq 3, defining two vectors: one to measure, $\Phi(k)$, and other parameters, $\Theta(k)$, then we have:

$$y(k) = \Phi^T(k)\Theta(k) + e(k)$$

(5)

Where:

$$\Phi^T(k) = \begin{bmatrix} -y(k-1) - y(k-2) ... \\ -y(k-na) u(k-d) ... u(k-d-nb) \end{bmatrix}$$

(6)
\[ \theta^T(k) = [a_1 a_2 a_3 ... a_m b_1 b_2 ... b_m] \]  

How are performed N measurements, which are used to determine \( a_i \) e \( b_j \), can be represented in matrix Eq. 7:

\[ Y = \phi \theta + E \]  

Or:

\[
\begin{bmatrix}
y(0) \\
y(1) \\
\vdots \\
y(N-1)
\end{bmatrix}
= 
\begin{bmatrix}
\phi^T(0) \\
\phi^T(1) \\
\vdots \\
\phi^T(N-1)
\end{bmatrix}
\begin{bmatrix}
\theta \\
e(0) \\
\vdots \\
e(N-1)
\end{bmatrix}
\]

It has to be the output vector is given by:

\[ Y^T = [y(0) y(1) y(2) ... y(N-1)] \]  

The estimated parameter vector \( \hat{\theta} \) can be obtained through the procedure of least squares. The best prediction of system output is given by:

\[ \hat{Y} = \phi \hat{\theta} \]  

and the prediction error is evaluated by:

\[ \varepsilon = Y - \hat{Y} \]  

The Markov estimator also called the weighted least squares estimator is obtained by minimizing the following criterion:

\[ J = \min_{\hat{\theta}} \left\| Y - \phi \hat{\theta} \right\|^2 \]  

One condition is that the matrix \( W \) is symmetric and positive:

\[ W = 
\begin{bmatrix}
W(0) & 0 & \cdots & 0 \\
0 & W(1) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & W(N-1)
\end{bmatrix} \]  

The elements of \( W \), \( W(i) \) are weighting each component of the error and depending on the accuracy of the measure. Derivative of the Eq.14 and equaling to zero, we have:

\[ \frac{\partial J^2}{\partial \hat{\theta}} = -2 \left( Y^T W \phi \right)^T + 2 \phi^T W \phi \hat{\theta} = 0 \]  

You can then calculate the least squares estimator:
\[ \hat{\theta} = \left[ \varphi^T \varphi \right]^{-1} \varphi^T Y \]  

(17)

Certain conditions must be met in order to have the minimum:

\[ \frac{\partial J^2}{\partial \hat{\theta}} = 2Y^T W\varphi > 0 \]  

(18)

- \( \varphi^T W\varphi \) should be a non-singular matrix;
- The vector \( \hat{\theta} \) can be obtained provided that \( \varphi^T W\varphi \) is a positive definite matrix;
- The system must be persistently excited to avoid the case of common lines in the matrix \( \varphi \).

The estimator of the non-recursive least square is obtained assuming that:

\[ \hat{\theta} = \left[ \frac{1}{\sigma^2} \left[ \varphi^T \varphi \right]^{-1} \varphi^T \right] Y \]

\[ \hat{\theta} = \left[ \varphi^T \varphi \right]^{-1} \varphi^T Y \]  

(19)

By applying the method of least squares is important to make some observations:

- There is a condition of excitement, in which the matrix \( \varphi^T \varphi \) is invertible;
- It is assumed that all measures have the same precision, because the least squares weights all errors;
- Estimated discrete transfer function, determination of the continuous transfer function is obtained by the relations of backward, forward or trapezoidal (Moraes, 2004).

Basically, the procedure for the estimates of non recursive least squares consists of injecting a signal in plant-type PRBS (Pseudo Random Binary Sequence) with amplitude \( \pm 1 \). This procedure is off-line. This condition is necessary in order to have sufficient signal excited. The output values are stored in an array to implement, immediately after, the equations for the method.

4. Data communication with Matlab

Matlab is reputable software for simulation, programming, widely used in the exact sciences, especially in teaching engineering. However, there was always a concern in academic power to transform the source code developed in Matlab for other languages like C++ and Delphi, although the otherwise it is not always observed. This is because applications developed in C language, for example, can act directly into the parallel port, serial and USB PC, making researches, in its implementation phase, developed in these languages programming. Notably, Matlab is used in much of cases, the simulation phase of a research project. Aspects the simulation are notable in the work-end courses in undergraduate courses or dissertations and theses in graduate school, where does not develop the trial. However, it is possible to Matlab to communicate with the ports on the PC using a program developed using C++ and Matlab function that allows interface with the program developed. Use the command "Mex" in order to do so. The sequence of procedures is as follows: first develops the program in C++ language to enable communication with the ports on the PC, Saving the same with the cpp extension. This program should be saved Matlab work folder and then performs the sequence of commands in Matlab to implement the control and / or outside readings.

Note that the possibilities are endless, as applications various control techniques (PID, fuzzy, etc.), identification systems and performance analysis in control systems, and for finally, compiles the cpp file, previously saved in the folder work with the command mex typed in command window. It necessary to define a compiler C++ previously installed on your PC to the achievement of the compilation process for this, it is used the command “mex - setup ”. For this work we used the visual C++. In this work we used the PC’s parallel port, for access to the external environment, which is
connected a circuit interfacing. The program, developed using C++ is intended to allow access and data exchange between a specific software and executable itself developed in C++ and a device connected to this Port.

Compilation process generates a file DLL, which is responsible for making Matlab to communicate with the ports on the PC (Carmo, 2008).

5. The Identification

The whole process of identification is based on the use of Matlab as a tool and a DLL developed in C++, to enable communication with Matlab's parallel port. The parallel port was used for an electronic circuit responsible for converting analog to digital and digital-analog. The communication and data sampling was performed with Simulink (Figure 4).

![Figure 4. Blocks used in Simulink for communication with the plant.](image)

To identify the transfer function of the plant, as well as the transfer function of the feedback, we used an artifice, which is injected a signal into the system in open loop and immediately after the closed loop system, since it is not had the transfer function of one of them, as in Figures 5 and 6.

![Figure 5. Open-loop identification.](image)
By applying the method we obtained two vectors which were applied the procedures described in section 2, coming to equations on the model ARMAX (Auto Regressive Moving Average Exogenous) (Aström, 2000):

\[ A(z^{-1})Y(z) = z^{-d}B(z^{-1})U(z) + E(z) \]  \hspace{1cm} (20)

Applying the \texttt{c2d} command of Matlab, and manipulating the functions obtained in closed loop and open loop, we can write transfers functions \( G(s) \) and \( H(s) \), or the plant itself and the transducer.

\[
G(s)H(s) = \frac{-3637s + 6.755E6}{s^2 + 8640s + 3.453E5} \]  \hspace{1cm} (21)

\[
G_{cl}(s) = \frac{2468s + 6.46E7}{s^2 + 2313s + 1.329E7} \]  \hspace{1cm} (22)

\[
G(s) = \frac{2468s^3 + 7.699E7s^2 + ...}{s^3 + 1.095E4s^2 + 3.36E7s^3 + ...} \]  \hspace{1cm} (23)
\[
\frac{3.41E11s + 4.59E14}{1.15E11s + 4.6E12} \]

\[
H(s) = \frac{-1.473s^3 - 670.7s^2 + ...}{s^3 + 3.119E4s^2 + ...} \]  \hspace{1cm} (24)
\[
\frac{1.32E7s + 3.63E10}{1.381E8s + 1.86E11} \]

Figure 7 shows the response of the transfer function in closed loop by a signal applied at the entrance. Note that the actual signal is very close to the signal identified.
Figure 7. a) Control Signal of the plant; b) system response and the actual transfer function identified.

6. Conclusions

Presented in this paper the method of least squares non-recursive identification model of a mechanical system, where dynamic characteristics have changed over time. We obtained responses that have made a commitment quite reliable compared to the real system, demonstrating the functionality of the method. It remains to be emphasized, however, it is necessary to verify that the transfer function is identified approximate second order so as to facilitate control design simpler, thus allowing a range as to what is proposed in this work, or allow use of such equipment for the laboratory control. This work, developed from the environment, will bring the possibility of teaching the techniques of least squares, in addition to developing the teaching of various mathematical modeling techniques such as the method of relay, the not symmetrical relay and even more advanced techniques such as of the instrumental variable. This new modeling approach for identification, as opposed to phenomenological modeling, brought the possibility of using the kit not only devoted to teaching techniques of control, but also the introduction to the mathematical modeling of systems.

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